Massive Parallelism in Economics and Climate Change

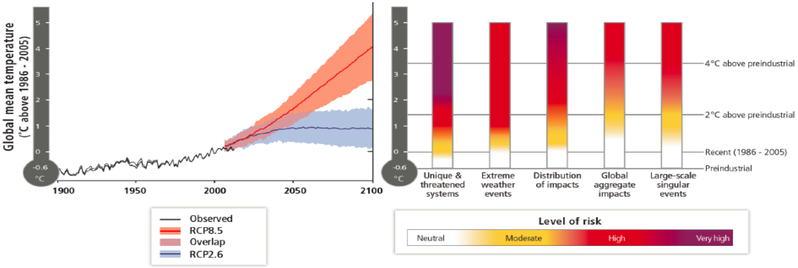
Yongyang Cai

Hoover Institution, Stanford University
Becker Friedman Institute, University of Chicago
Center for Robust Decision Making on Climate and Energy Policy, University of Chicago

August 11, 2014

Global Warming

Projected Temperature from the Fifth Assessment Report of the United Nations Intergovernmental Panel on Climate Change (IPCC 2014):

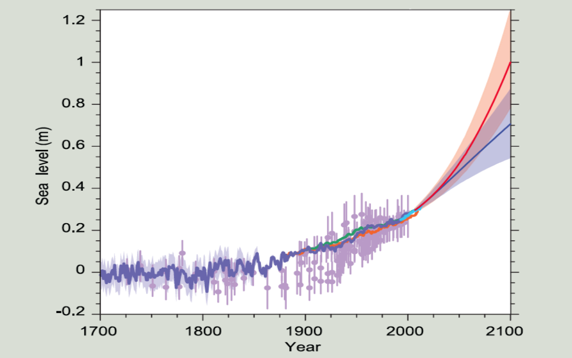


- Current damage: about 1.6% loss of global GDP annually
- In 2030: 3.2% loss of global GDP
  - the world's least developed countries will suffer losses of up to 11% of their GDP
  - 2% of the GDP of the US
  - $1.2 trillion for China

Abrupt, Stochastic, and Irreversible Climate Change

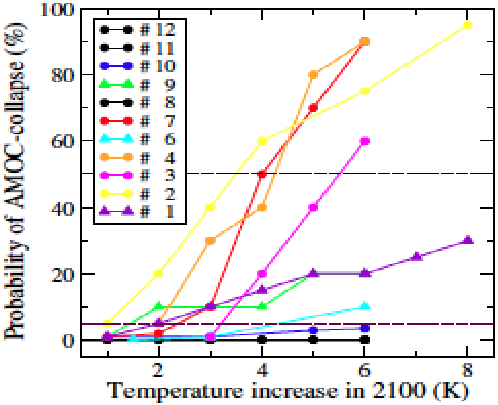
Tipping element: a significant event with permanent damages

- Antarctic and Greenland ice sheet melting (sea level rise, IPCC 2014)
  - 0.2% ~ 4.6% of the world's population is flooded annually in 2100
  - 0.3% ~ 9.3% loss of the global GDP annually in 2100



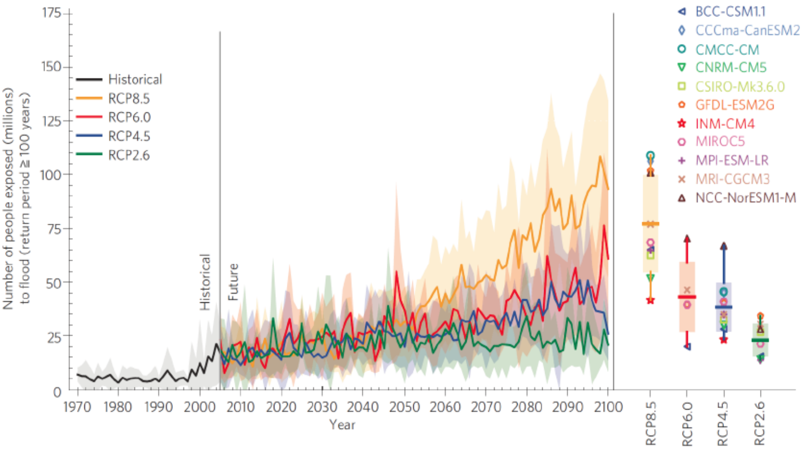
Abrupt, Stochastic, and Irreversible Climate Change

- Thermohaline circulation collapse (THC)
  - Experts' subjective probability of AMOC (Atlantic Meridional Overturning Circulation) collapse in 2100 (Zickfel et al. 2007)



Risk Uncertainty

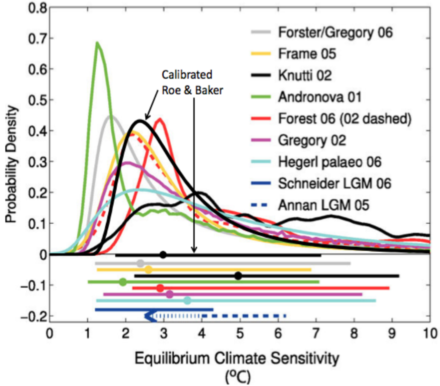
- taste shocks, uncertain technological advances (e.g., quantum computers), financial crisis, weather shocks (IPCC 2014)



Parameter Uncertainty

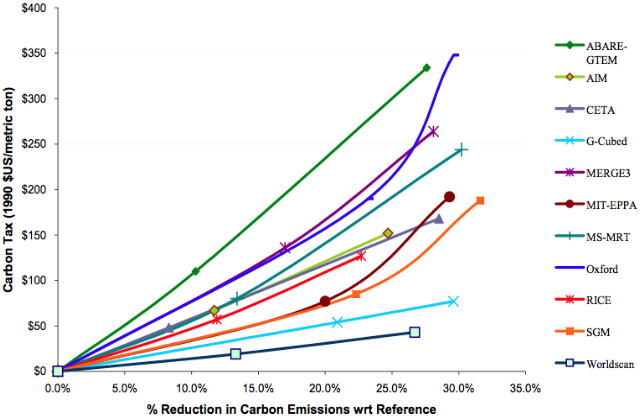
- policymakers do not know parameters that characterize the economic and/or climate systems
  - Climate sensitivity: the increase of surface temperature from a doubling of carbon concentration in the atmosphere
  - Distribution of climate sensitivity (United States International Working Group on Social Cost of Carbon, 2010)

Figure 2: Estimates of the Probability Density Function for Equilibrium Climate Sensitivity (°C)



Model Uncertainty

- Model uncertainty: policymakers do not know the proper model or the stochastic processes
  - EMF-16 model predictions of marginal abatement costs (Fischer and Morgenstern, 2005)

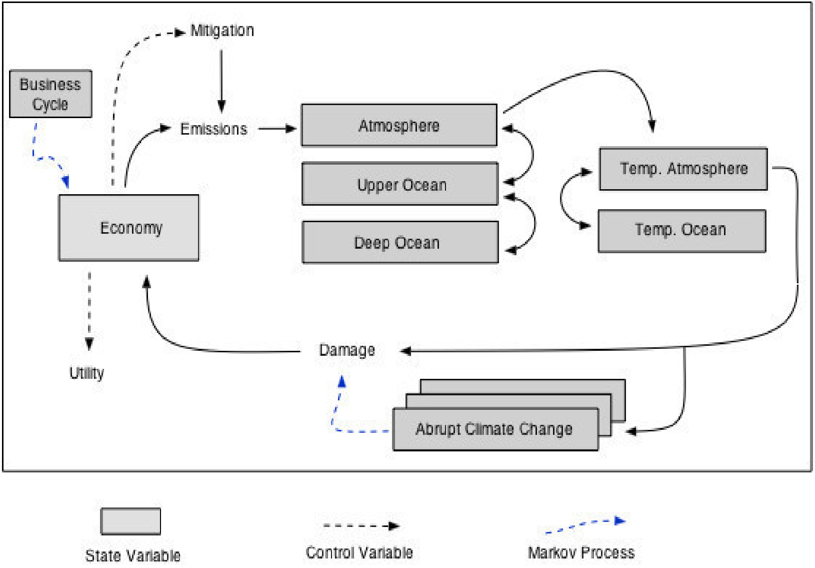


Climate Change Policy Analysis

Question: What can and should be the policy response to rising CO2 concentrations in the face of uncertainty?

- Economists analyze simple stylized models of pieces of the system
  - Pencil and paper preferred to computers and code
  - Deterministic model: economic actors know perfectly future economic and climate events
  - Myopic: ignore trend and dynamics of systems
- We are trying to change that
  - Create dynamic and stochastic integrated models of climate and economy (DSICE)
    - uncertain economic growth with long-run risk
    - climate tipping risk
    - parameter uncertainty and learning
    - flexible preferences regarding uncertainty
  - Create robust and general tools that can use state-of-the art numerical methods on modern computer architectures: Climate change policy is the application

DSICE Framework



DSICE with Epstein-Zin Preferences

- Epstein-Zin Preferences: recursive utility function (distinguish between risk aversion and the desire for consumption smoothing)
  - ψ: intertemporal elasticity of substitution – "consumption flexibility"
  - γ: risk aversion parameter
- Nine-dimensional state vector: $s = (K, \mathbf{M}, \mathbf{T}, \zeta, \chi, J)$
  - K: capital; $\mathbf{M} = (M_{\mathrm{AT}}, M_{\mathrm{UO}}, M_{\mathrm{LO}})$; $\mathbf{T} = (T_{\mathrm{AT}}, T_{\mathrm{OC}})$; ζ: uncertain technology growth; χ: long-run risk in economic growth; J: climate tipping state
- Bellman equation (a.k.a. Hamilton-Jacobi-Bellman PDE equation for the continuous time) for the dynamic stochastic problem:

$$
\begin{aligned}
V_t(s) = \max_{C,\mu} \quad & u_t(C_t, L_t) + \beta \left[\mathbb{E}_t \left\{ \left(V_{t+1}\left(s^+\right)\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right\} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}, \\
\text{s.t.} \quad & K^+ = (1-\delta)K_t + \mathcal{Y}_t(k, T_{\mathrm{AT}}, \mu, \zeta, J) - C_t, \\
& \mathbf{M}^+ = \Phi^{\mathrm{M}}\mathbf{M} + \left(\mathcal{E}_t\left(K, \mu, \zeta\right), 0, 0\right)^\top, \\
& \mathbf{T}^+ = \Phi^{\mathrm{T}}\mathbf{T} + \left(\xi_1 \mathcal{F}_t\left(M^{AT}\right), 0\right)^\top, \\
& \zeta^+ = g_\zeta(\zeta, \chi, \omega_\zeta), \\
& \chi^+ = g_\chi(\chi, \omega_\chi), \quad J^+ = g_J(J, \mathbf{T}, \omega_J)
\end{aligned}
$$

Numerical Dynamic Programming

- Numerical Dynamic Programming Algorithm:
  - Initialization. Choose the approximation grid, $\mathbb{S} = \{s_i : 1 \leq i \leq m\}$, and choose functional form for $\hat{V}(s; b)$. Let $\hat{V}(s; b_T) = V_T(s)$. Iterate through steps 1 and 2 over $t = T - 1, ..., 1, 0$.
  - Step 1. Maximization step (in parallel): Compute

    $$v_i = \max_{a_i \in \mathcal{D}(x_i, t)} u_t(s_i, a_i) + \beta \mathbb{E}\{\hat{V}(s_i^+; b_{t+1})\},$$

    for each $x_i \in X$, $1 \leq i \leq m$.
  - Step 2. Fitting step: Using the appropriate approximation method, compute the b_t such that $\hat{V}(s; b_t)$ approximates (s_i, v_i) data.
- Three main computational parts: optimization, integration, and approximation

Parallelization of DSICE

- Discretized dimensions (ζ, χ, J): $91 \times 19 \times 16 = 27,664$ points
- Six-dimensional continuous states $(k, \mathbf{M}, \mathbf{T})$: 56K approximation nodes per discrete point
- Master-Submaster-Worker system
  - use the Cartesian virtual topology for communicator
  - dynamic load balancing
- Total number of optimization problems: 372 billion

Num of Cores	Wall Clock Time	Total CPU Time
69,184	11.2 hours	88 years

Parallelization of Uncertainty Quantification in DSICE

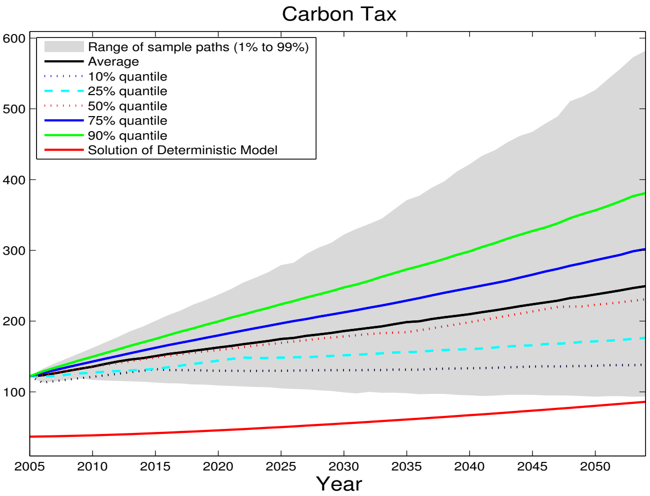
- Six uncertain parameter values
  - intertemporal elasticity of substitution
  - risk aversion
  - hazard rate of tipping
  - expected damages
  - variance of damages
  - expected duration of the tipping process
- Master-Submaster-Worker system
- Solve on grids in parameter space (2,430 cases)

Num of Cores	Wall Clock Time	Total CPU Time
8,160	1.04 hour	0.97 year

Social Cost of Carbon for DSICE with Tipping

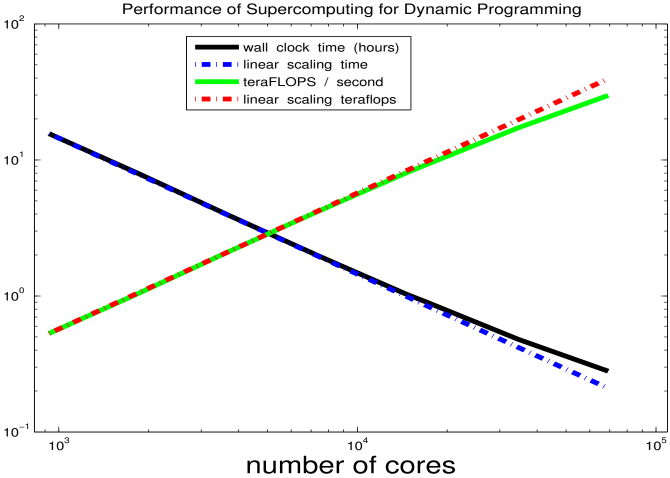
Hazard Rate Parameter	Mean Damage Level	Expected Duration	Relative Variance	SCC			
				$\psi = 0.5$		$\psi = 1.5$	
				$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
0.0045	10%	5	0%	88	109	386	480
			40%	91	140	400	586
		200	0%	54.4	57.8	227	259
			40%	54.9	61.5	232	306
0.0025	10%	5	0%	67	83	274	364
			40%	69	103	285	467
		200	0%	47.2	49.6	174	195
			40%	47.5	51.9	176	224

Dynamics of Solutions for DSICE



- Optimal Initial Carbon Tax: 125 US$/tC (deterministic model: 37 US$/tC)

Scalability in Parallel Dynamic Programming



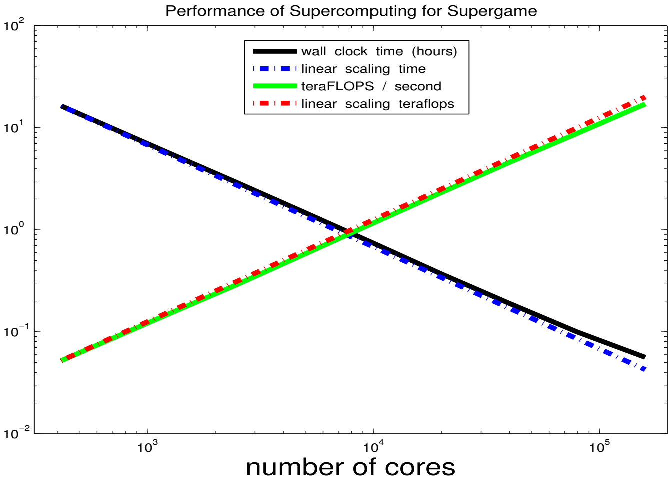
#core	Wall Clock Time (hour)	teraFLOPS / sec
928	15.6	0.53
69,184	0.28	29.79

Parallelization in Dynamic Games

- Social decisions are dynamic games
  - Economic policy analysis focuses mainly on what is "optimal".
  - Real decisions are made by people acting within a social system with procedures and rules: a game
- Total number of optimization problems: 1.4 trillion

Num of Cores	Wall Clock Time	Total CPU Time
100,000	1.8 hours	21 years

Scalability in Parallel Dynamic Games



#core	Wall Clock Time (hours)	teraFLOPS / sec
416	16.36	0.052
159,744	0.0561	17.07

Summary

- We construct a DSICE model that incorporates a level of uncertainty for the economy and the climate supported by data
- We find a far broader range of possible carbon taxes, and show that there is a good chance that the social cost of carbon could be far larger than currently thought
- We use the most advanced mathematical methods and computer hardware to solve the problems
- Our software has good scalability for supercomputing

Acknowledgement

- Kenneth Judd and Thomas Lontzek (collaborators)
- Lars Hansen and other colleagues in RDCEP
- Blue Waters and Beagle supercomputers
- NSF
- Paul Messina and ATPESC
- Future technical development for DSICE:
  - ADLB, PETSc, Communication-Avoiding Algorithms, Parallel Optimization (TAO), HDF5, MPI+OpenMP+GPU, etc.