Nonparametric Reconstruction of the Dark Energy Equation of State

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Literature

This talk is based on:

- T. Holsclaw, U. Alam, B. Sanso, H. Lee, K. Heitmann, S. Habib, D. Higdon, ***"Nonparametric Reconstruction of the Dark Energy Equation of State"***, Phys. Rev. D82, 103502 (2010)
- T. Holsclaw, U. Alam, B. Sanso, H. Lee, K. Heitmann, S. Habib, D. Higdon, ***"Nonparametric Dark Energy Reconstruction from Supernova Data"***, Phys. Rev. Lett., 241302 (2010)
- T. Holsclaw, U. Alam, B. Sanso, H. Lee, K. Heitmann, S. Habib, D. Higdon, ***"Nonparametric Reconstruction of the Dark Energy Equation of State from Diverse Data Sets"***, Phys. Rev. D84, 083501 (2011)
- T. Holsclaw, B. Sanso, H. Lee, D. Higdon, K. Heitmann, U. Alam, S. Habib, ***"Gaussian Process Modeling of Derivative Curves"***, Technometrics (2012)

Recent Follow-up Work by Other Groups:

- A. Shafiello, A. Kim, E. Linder, ***"Gaussian Process Cosmography"***, Phys. Rev. D82, 123530 (2012)
- M. Seikel, C. Clarkson, M. Smith, ***"Reconstruction of Dark Energy and Expansion Dynamics using Gaussian Processes"***, JCAP 1206, 036 (2012)

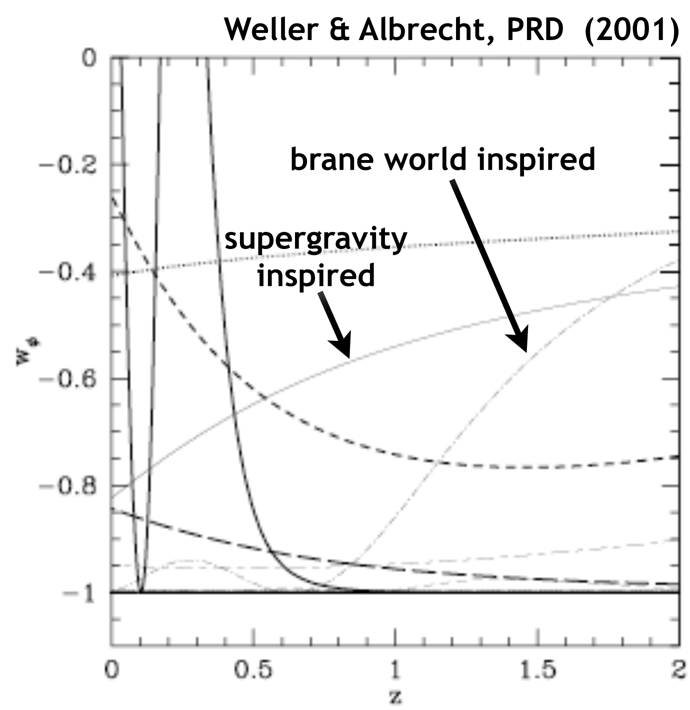
Introduction to Gaussian Processes:

- C.E. Rasmussen, K.I. Williams, MIT Press, ***"Gaussian Processes for Machine Learning"***, www.GaussianProcess.org/gpml

For more application examples for GPs in cosmology, talk to us…

A Dynamical Origin of Dark Energy?

- **Morning talk:** Origin of dark energy is great mystery, *SNe provide excellent probe to investigate nature of dark energy*
- **What do we do if we don't know what to do?**
  - We invent a scalar field ϕ
  - Key: we (theorists) predict that for a "physically well motivated model" the EOS w(z) should be time varying
  - Observers have something to look for!
  - Problem: endless possibilities to invent models, see examples on the right
- **Develop a model-independent reconstruction scheme for w(z)**
- **Afternoon talk 1:** *SNe may have some issues...*



Equation of state parameter w as a function of redshift for various models

A Dynamical Origin of Dark Energy?

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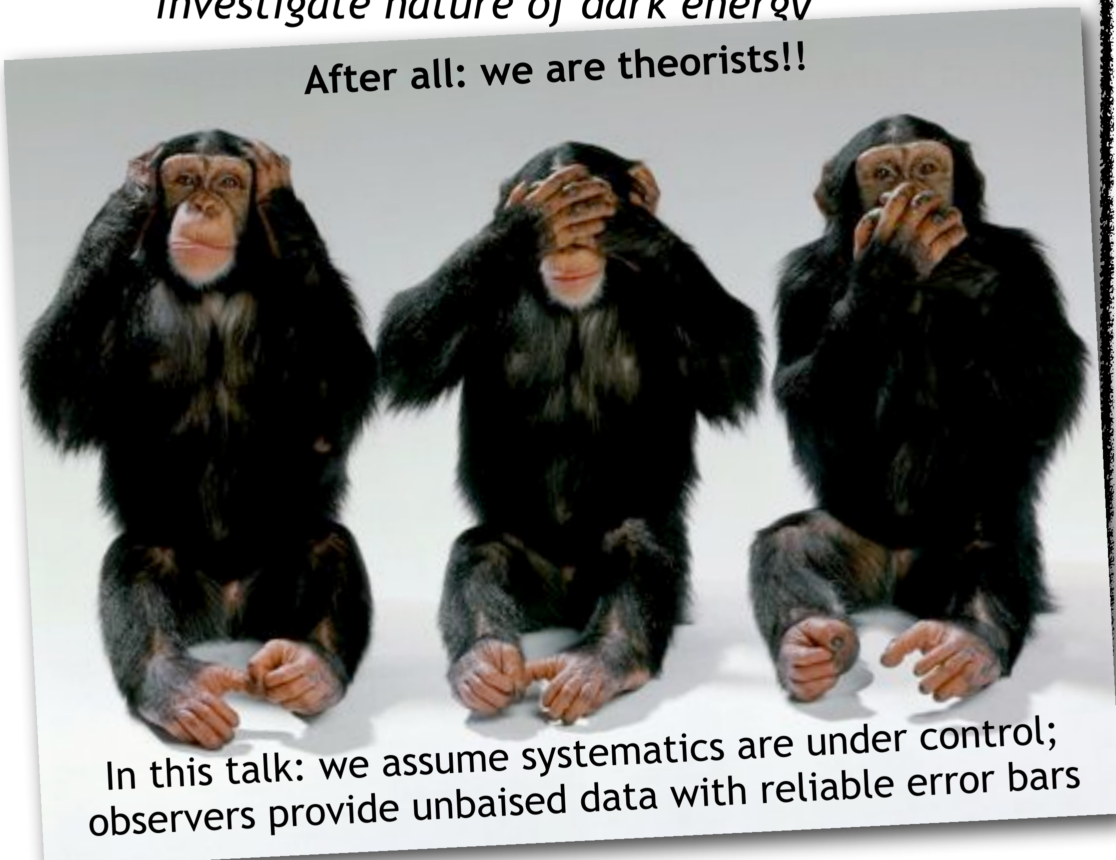
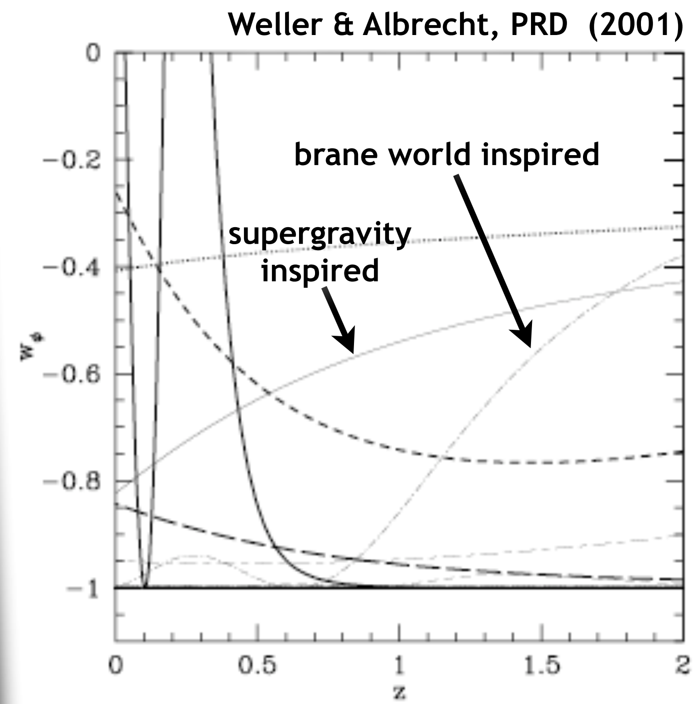
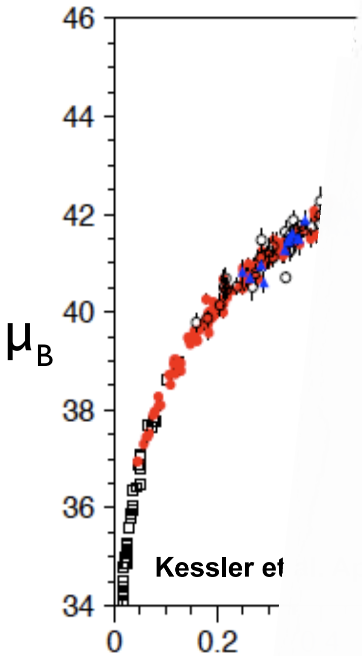
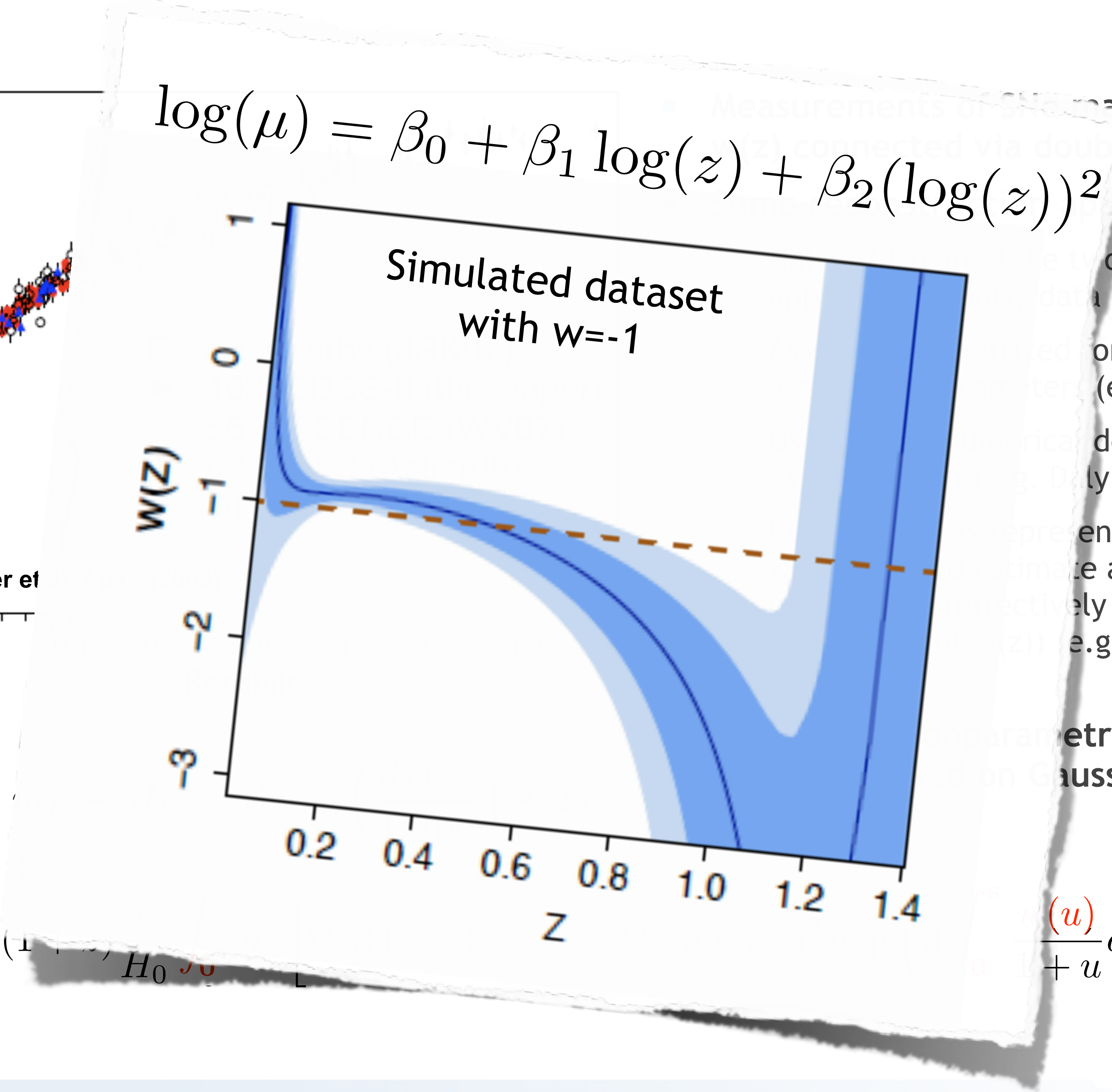




Equation of state parameter w as a function of redshift for various models

The Reconstruction Task



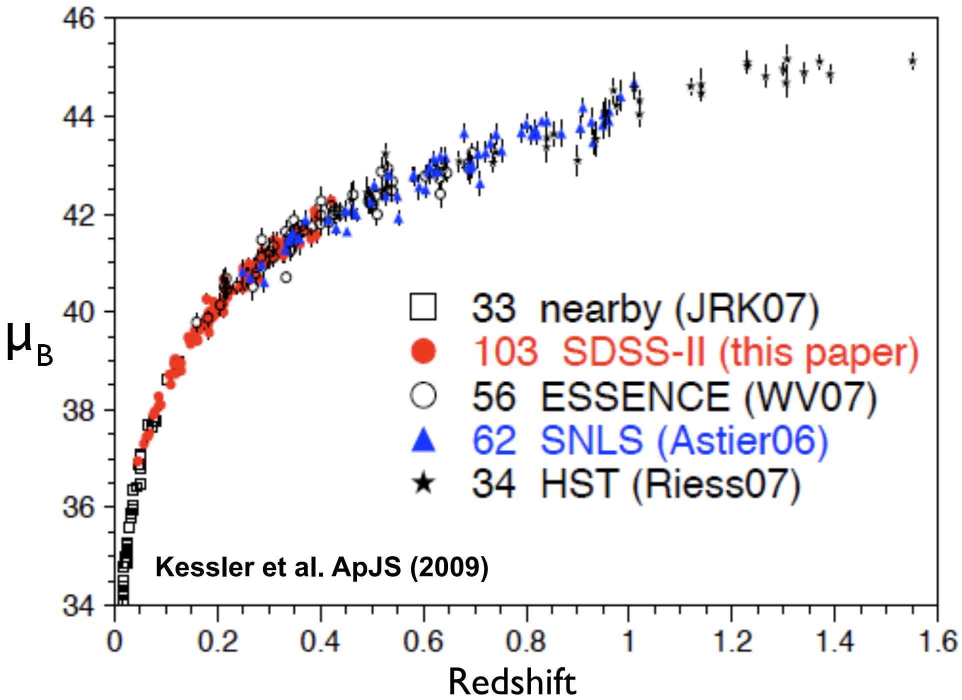


- …agnitudes and …e-integral
- …roaches:
  - …o derivatives, bad
  - …orm for w, estimate …(e.g. Linder 2003)
  - …derivatives to …ly & Djorgovski 2003)
  - …entation for w(z) (bins, …e associated …ly piecewise constant …e.g. Huterer & Cooray
- …etric reconstruction …aussian Process

$\mu_B(z) = \ldots$

$d_L(z) = (1+z)\frac{c}{H_0}\int_0 \ldots \frac{(u)}{+u}du\Big)\Big]^{-\frac{1}{2}}$

The Reconstruction Task

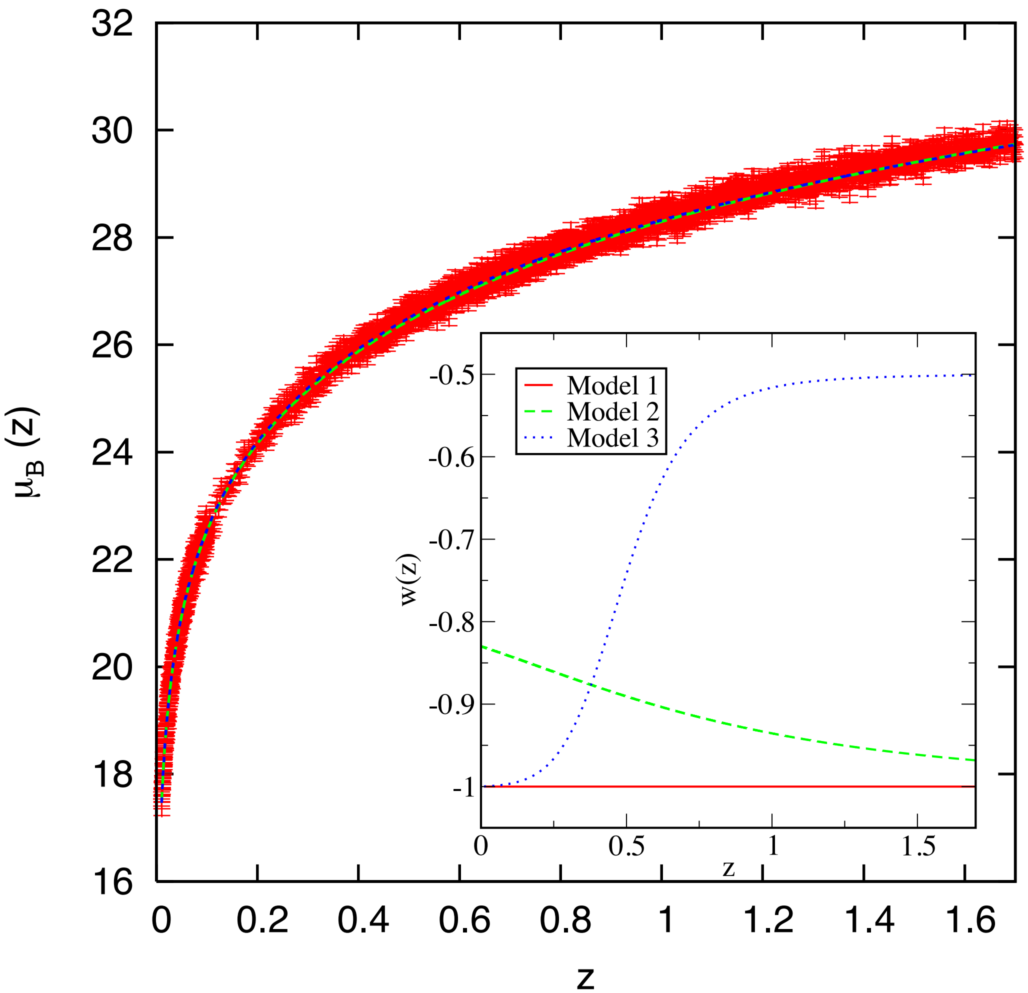


$$\mu_B(z) = m_B - M_B = 5\log_{10}\left(\frac{d_L(z)}{1\text{Mpc}}\right) + 25$$

- **Measurements of SNe magnitudes and w(z) connected via double-integral**
- **Some reconstruction approaches:**
  - Naive: fit μ and take two derivatives, bad approach for noisy data
  - Assume parametrized form for w, estimate associated parameters (e.g. Linder 2003)
  - Use filtered numerical derivatives to estimate w(z) (e.g. Daly & Djorgovski 2003)
  - Pick local basis representation for w(z) (bins, wavelets) and estimate associated coefficients (effectively piecewise constant description of w(z)) (e.g. Huterer & Cooray 2005)
- **Here: new, nonparametric reconstruction approach based on Gaussian Process models**

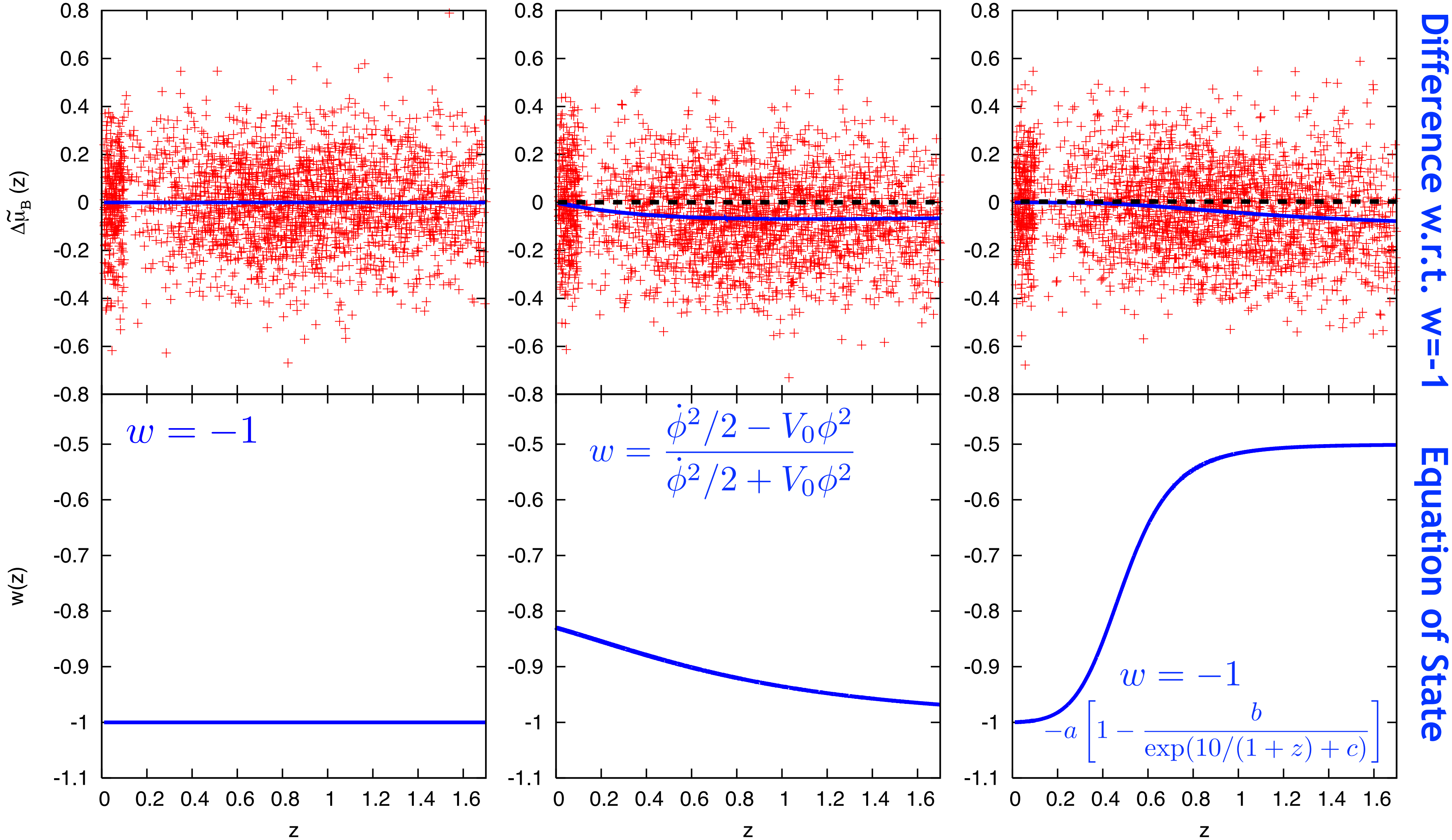
$$d_L(z) = (1+z)\frac{c}{H_0}\int_0^z ds\left[\Omega_m(1+s)^3 + (1-\Omega_m)(1+s)^3\exp\left(3\int_0^s \frac{w(u)}{1+u}du\right)\right]^{-\frac{1}{2}}$$

The Challenge



- Differences in the distance modulus μ are very small for different dynamical dark energy models
- To test our new method and compare with other methods we set up space mission quality dataset for three different dark energy models
- Assume a sample of 300 low redshift (z<0.1) SNe and 2000 more up to z=1.7, peak at z~0.8
- Compare with results from two parameterized models

The Models



$\Delta\tilde{\mu}_B$ (z)
w(z)
z
$w = -1$
$w = \frac{\dot{\phi}^2/2 - V_0\phi^2}{\dot{\phi}^2/2 + V_0\phi^2}$
$w = -1 - a\left[1 - \frac{b}{\exp(10/(1+z)+c)}\right]$
Difference w.r.t. w=-1
Equation of State

Our Approach: Gaussian Process Modeling

- **Nonparametric regression scheme, particularly well suited for interpolation of smooth functions**
- **Extending the notion of a Gaussian distribution over scalar or vector random variables into function space**

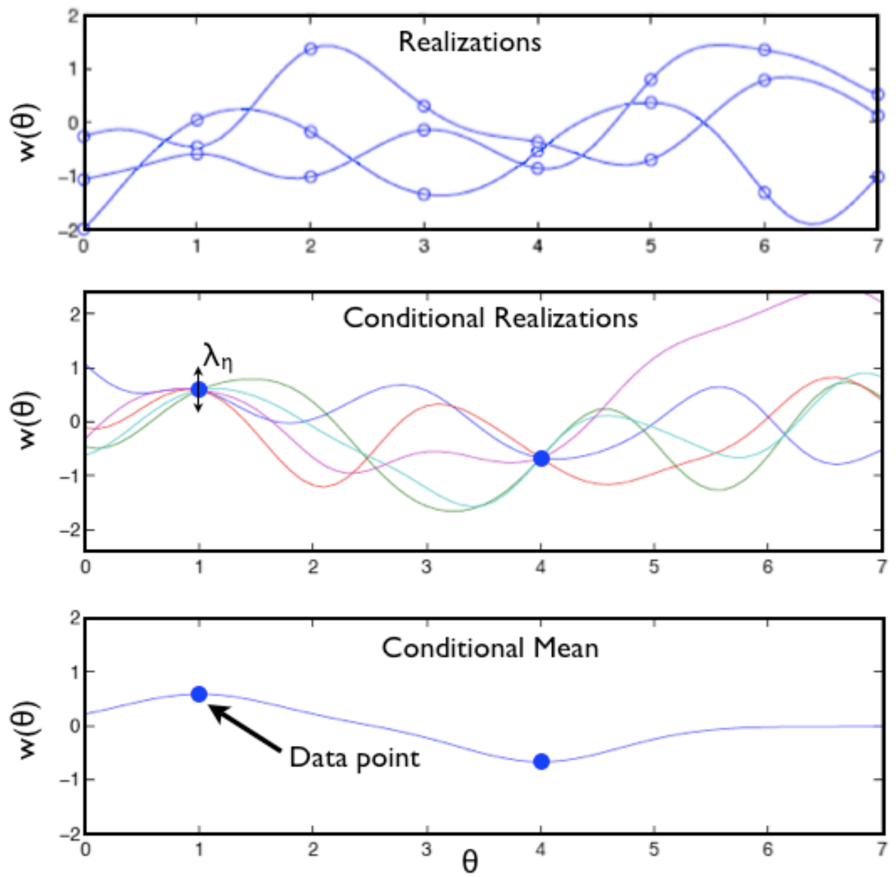
$$\mathbf{f} = (f_1, \ldots, f_n)^T \sim \mathcal{N}(\mu, \Sigma)$$

$$\Downarrow$$

$$f(\mathbf{x}) \sim \mathrm{GP}(\mu(\mathbf{x}), K(\mathbf{x}, \mathbf{x}')),$$

$$K(\mathbf{x}, \mathbf{x}') = cov(f(\mathbf{x}), f(\mathbf{x}'))$$

- **Correlation function critical to GP approach, attributes:**
  - Unity for **x=x'**, so replica are perfectly correlated
  - Large for **x≈x'**, high correlation of nearby points
  - Small for **x** far from **x'**, small correlation



Unconditioned GP:

$$w \sim N(0, \lambda_w^{-1} R)$$

$$R_{ij} = \exp\{-||\theta_i - \theta_j||^2\}$$

Conditioned GP:

$$\begin{pmatrix} \theta \\ \theta^* \end{pmatrix} \sim N\left(0, \begin{pmatrix} K & K_*^T \\ K_* & K_{**} \end{pmatrix}\right)$$

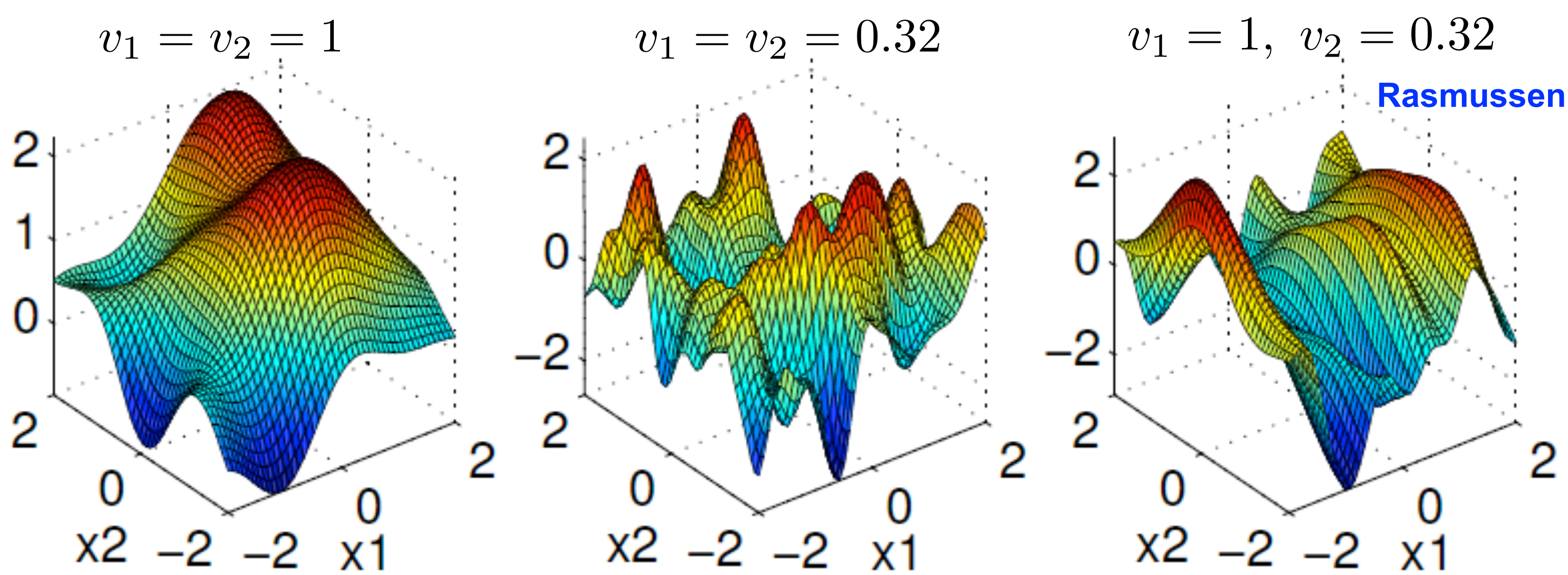
$$\theta_* | \theta \sim N(K_* K^{-1} \theta, K_{**} K^{-1} K_*^T)$$

Interlude: Gaussian Process Modeling

- Covariance function (symmetric, positive semi-definite) is the key ingredient in GP modeling. Depending on the application, various choices of the covariance function are possible, both in terms of form and underlying parameters
- **Commonly used: squared exponential:**

$$K(\mathbf{x}, \mathbf{x}') = v_0^2 \exp\left(-\sum_{d=1}^{D} \frac{(x_d - x'_d)^2}{2v_d^2}\right)$$

Hyperparameters: $\theta = (v_0, v_1, ..., v_D, \sigma_n^2)$



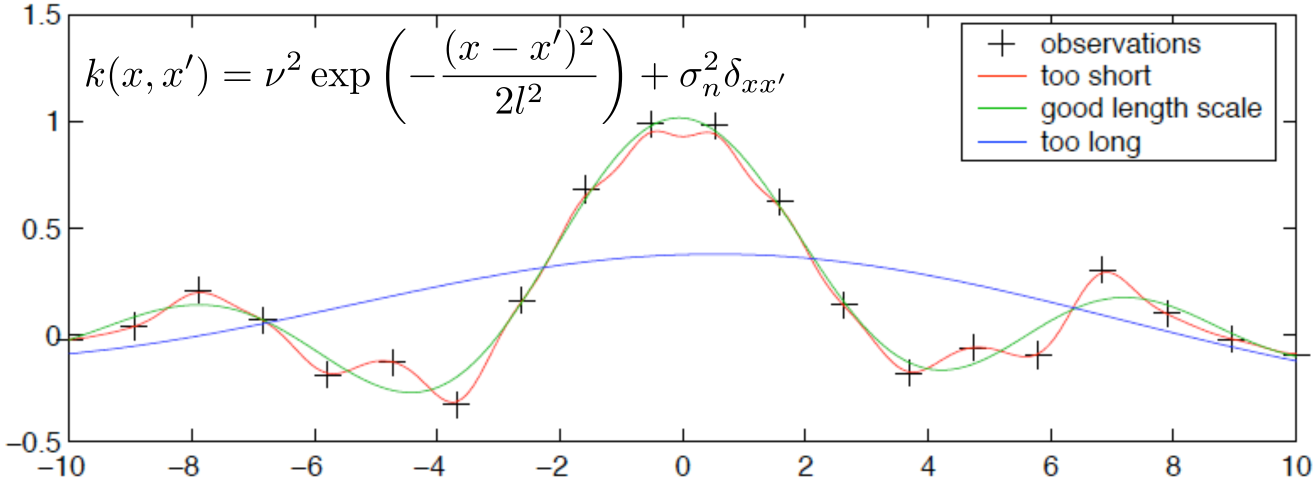
Rasmussen 2006

Interlude: Gaussian Process Modeling

- Model selection: choose a covariance function and determine the hyperparameters of the covariance functions. Usually, one has access only to noisy versions of the GP function draws ("observations") $y = f(\mathbf{x}) + \epsilon$. If the noise is independent, identically distributed with variance σ_n^2, then the prior on the observation is

$$cov(y_i, y_j) = k(y_i, y_j) + \delta_{ij}\sigma_n^2$$

- Hyperparameters are set by maximizing the marginal likelihood



Rasmussen 2006

Reconstructing w(z) with GP Modeling

- Assume a GP for dark energy equation of state parameter

$$w(u) \sim \mathrm{GP}(-1, K(u,u')), \quad K(u,u') = \kappa^2 \rho^{|u-u'|^\alpha}$$

- Need to integrate over this in the expression for the distance modulus, where

$$y(s) = \int_0^s \frac{w(u)}{1+u} du$$

- Use that the integral over a GP is another GP and specify covariance

$$y(s) \sim \mathrm{GP}\left(-\ln(1+s), \kappa^2 \int_0^s \int_0^{s'} \frac{\rho^{|u-u'|^\alpha} du du'}{(1+u)(1+u')} \right)$$

- A joint GP for the two variable can be constructed

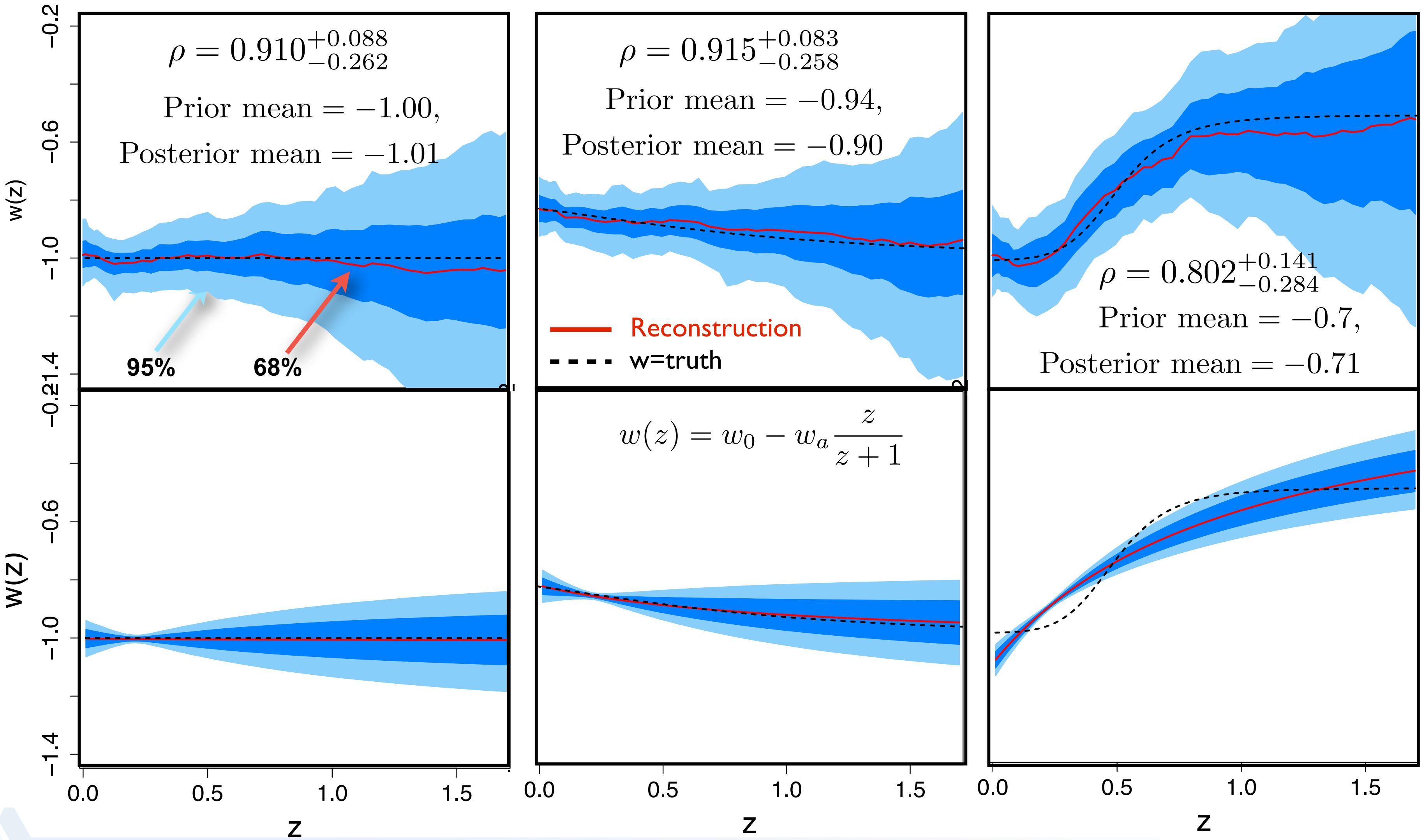
$$\begin{bmatrix} y(s) \\ w(u) \end{bmatrix} \sim \mathrm{GP}\left[\begin{bmatrix} -\ln(1+s) \\ -1 \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \right]$$

Reconstructing w(z) with GP Modeling

- where: $$\Sigma_{11} = \kappa^2 \int_0^s \int_0^{s'} \frac{\rho^{|u-u'|} du du'}{(1+u)(1+u')},$$ $$\Sigma_{22} = \kappa^2 \rho^{|u-u'|},$$ $$\Sigma_{12} = \Sigma_{21} = \kappa^2 \int_0^s \frac{\rho^{|u-u'|} du}{(1+u)}$$
- The mean is given by: $$\langle y(s)|w(u)\rangle = -\ln(1+s) + \Sigma_{12}\Sigma_{22}^{-1}\left[w(u) - (-1)\right].$$
- Now estimate correlation hyperparameters, ρ and κ, the GP points w(u), and any cosmological parameters jointly to reconstruct w(z)

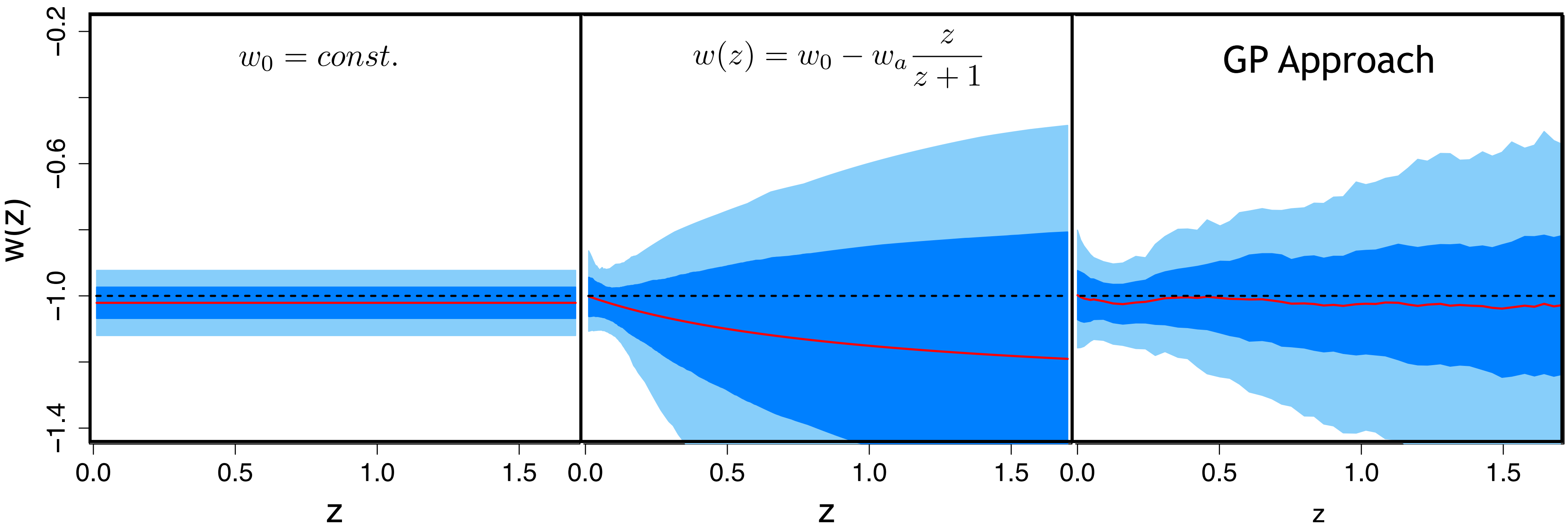
Results

- Simplify task by fixing $\Omega_m = 0.27$ and $\Delta_\mu = 0$
- GP model: $w(u) \sim \mathrm{GP}(-1, K(u,u'))$ with $K(z,z') = \kappa^2 \rho^{|z-z'|}$

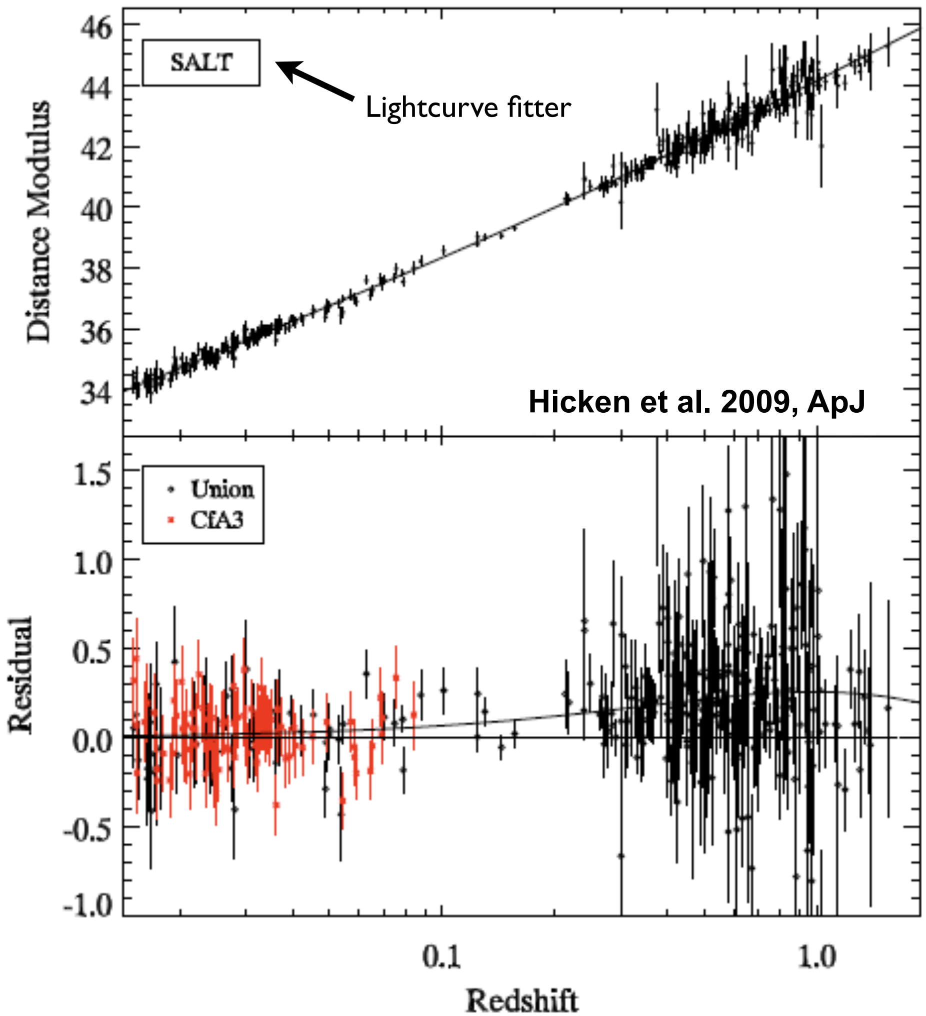


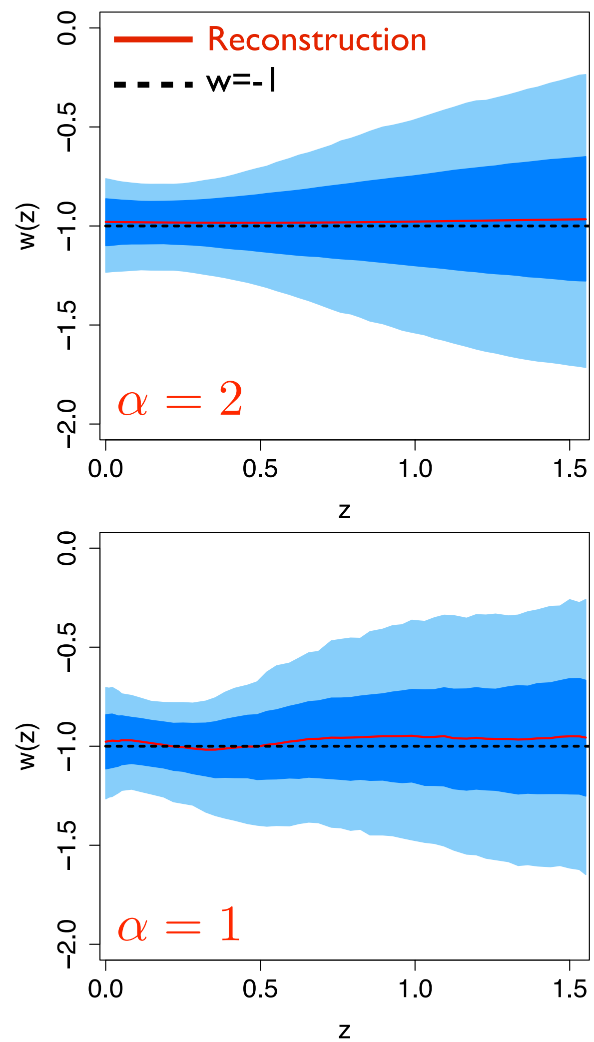
Results

- **Keep** Ω_m, Δ_μ **free this time, compare results for** ***w=const*** **data set**



Result for Recent Data





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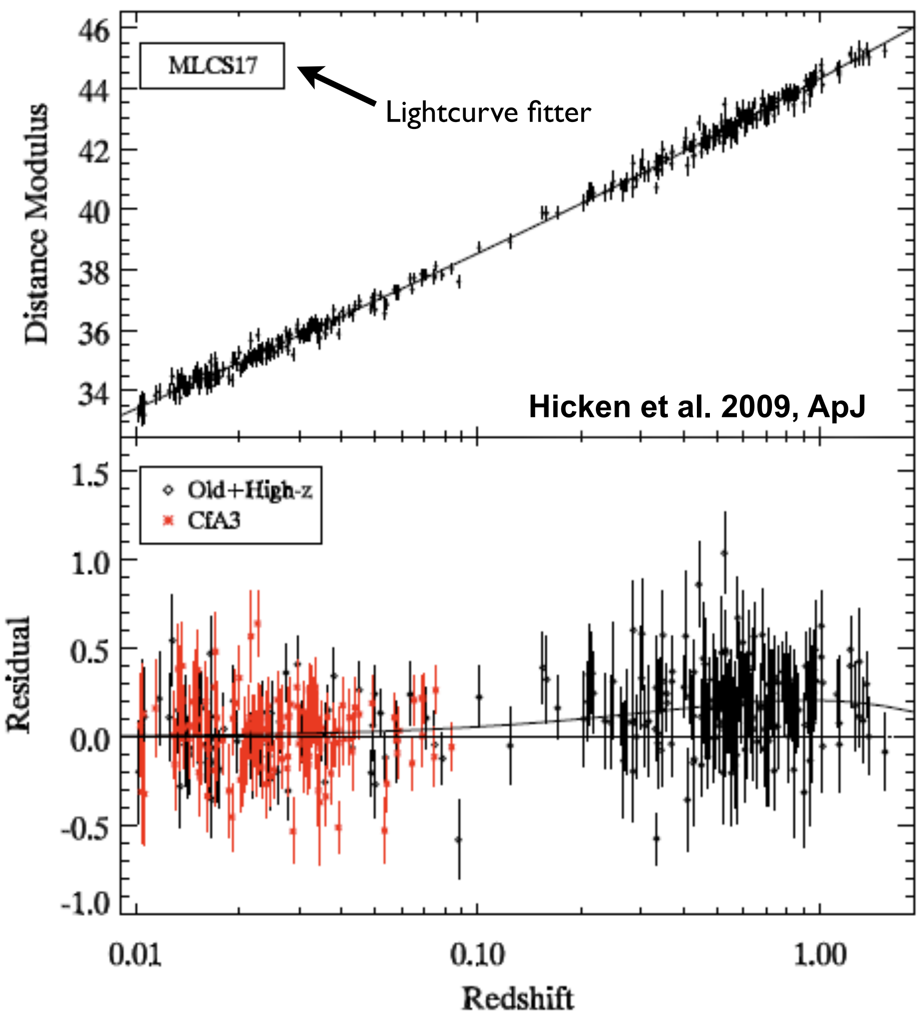
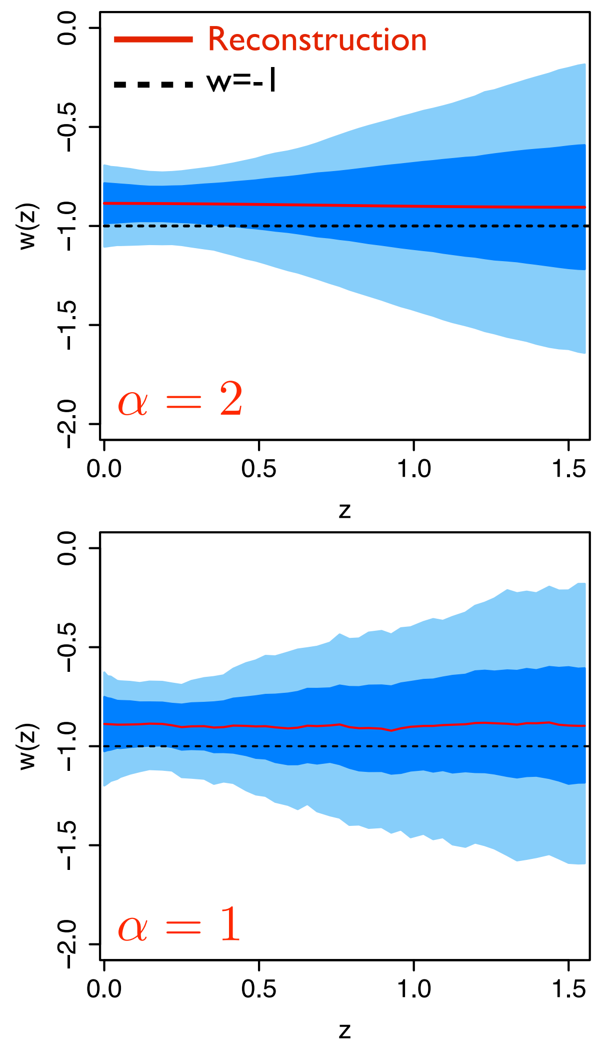
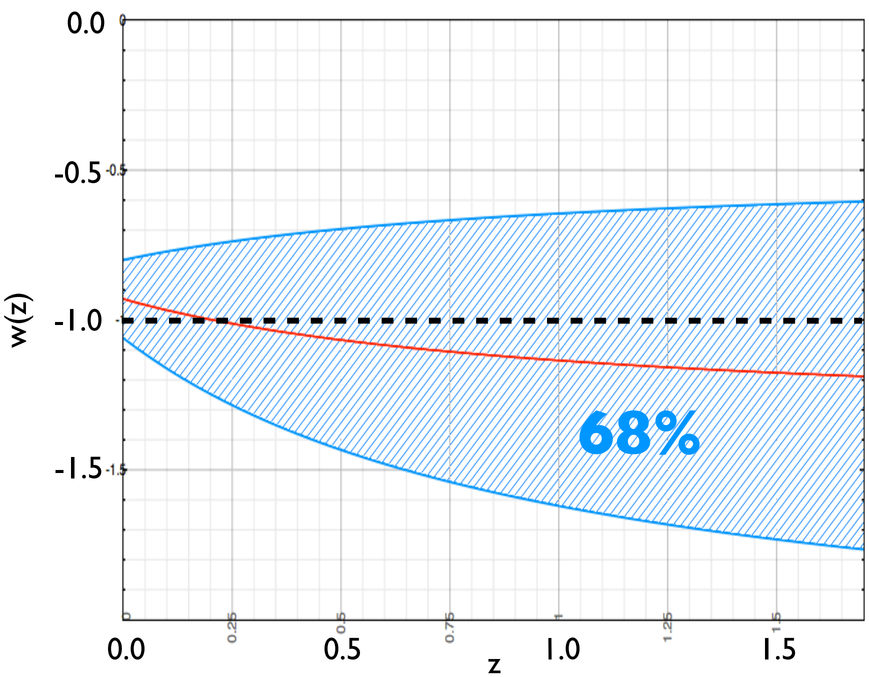
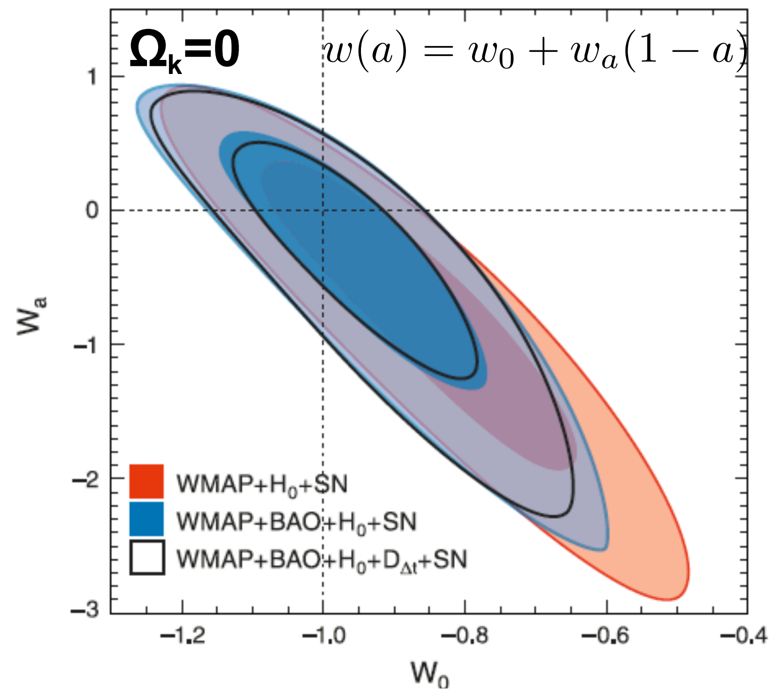




WMAP Team Analysis





Curvature	Parameter	$+$BAO$+H_0$	$+$BAO$+H_0+D_{\Delta t}$[a]	$+$BAO$+$SN[b]
$\Omega_k = 0$	Constant w	-1.10 ± 0.14	-1.08 ± 0.13	-0.980 ± 0.053
$\Omega_k \neq 0$	Constant w	-1.44 ± 0.27	-1.39 ± 0.25	$-0.999^{+0.057}_{-0.056}$
	Ω_k	$-0.0125^{+0.0064}_{-0.0067}$	$-0.0111^{+0.0060}_{-0.0063}$	$-0.0057^{+0.0067}_{-0.0068}$
		$+H_0+$SN	$+$BAO$+H_0+$SN	$+$BAO$+H_0+D_{\Delta t}+$SN
$\Omega_k = 0$	w_0	-0.83 ± 0.16	-0.93 ± 0.13	-0.93 ± 0.12
	w_a	$-0.80^{+0.84}_{-0.83}$	$-0.41^{+0.72}_{-0.71}$	$-0.38^{+0.66}_{-0.65}$

From: Komatsu et al. 2010, arXiv:1001.4538

Conclusion

- **Without having a clue what the origin of dark energy is, our approach to characterize it via its EOS w(z) has to be as general as possible -- nonparametric reconstruction**
- **Gaussian Process Modeling is a powerful tool for the task!**
- **The GP as used here has many useful features:**
  - The data is not massaged in any way
  - Robustness of results to GP hyperparameters can be easily tested
  - Data can be non-uniform in redshift
  - Easy extension to include diverse datasets (done in Holsclaw et at. 2011)
- **GP can be used for many other purposes:**
  - To build prediction schemes (we call those “emulators”) for different statistics, e.g. mass function or power spectrum from a small set of simulations
    1. GP used for interpolation of different models
  - Photometric redshift prediction
    1. Build training set from spectroscopic measurements
    2. Use GP approach as “machine learning” approach