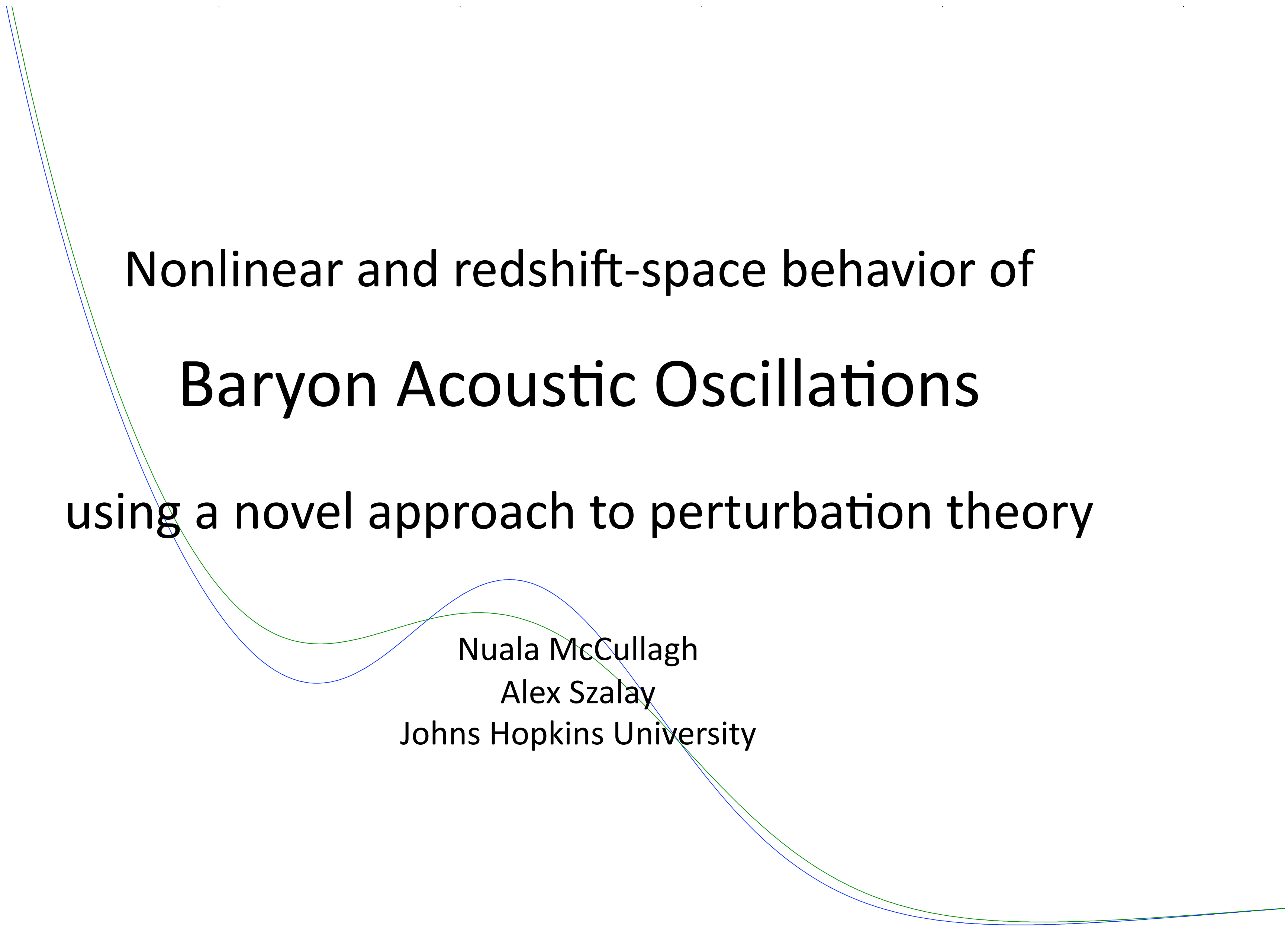
Nonlinear and redshift-space behavior of Baryon Acoustic Oscillations using a novel approach to perturbation theory



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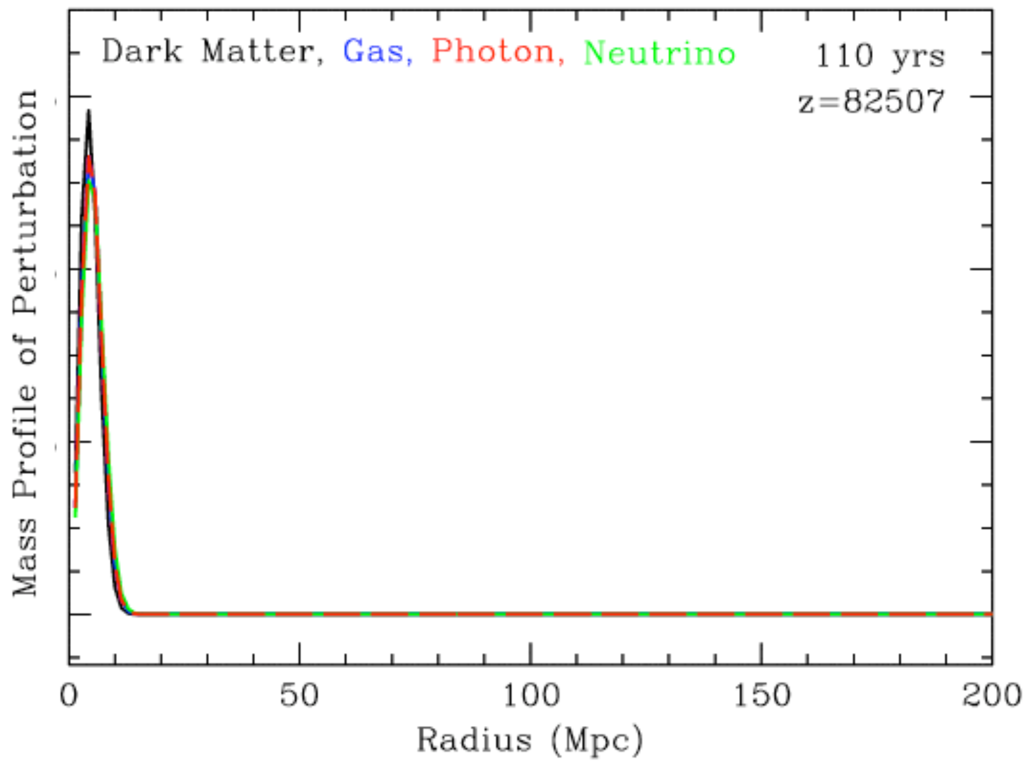
Outline

- Motivation
  - background of BAO as a “standard ruler”
  - modeling the evolution of acoustic peak allows us to accurately constrain cosmological parameters
    - nonlinearity
    - redshift-space distortions
- Previous work on nonlinearity and redshift-space distortions
  - standard perturbation theory
- New approach to perturbation theory in configuration space
  - motivation
  - results
  - directions for future research

BAOs Imprinted on the Matter Density



Eisenstein, http://cmb.as.arizona.edu/~eisenste/acousticpeak/

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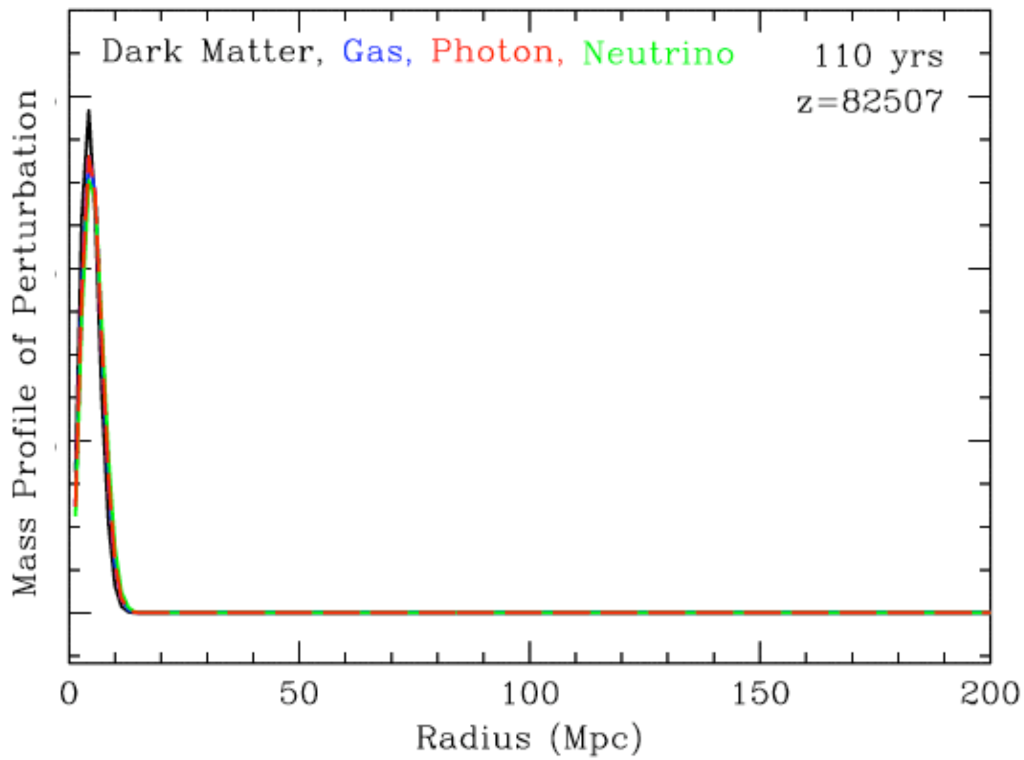

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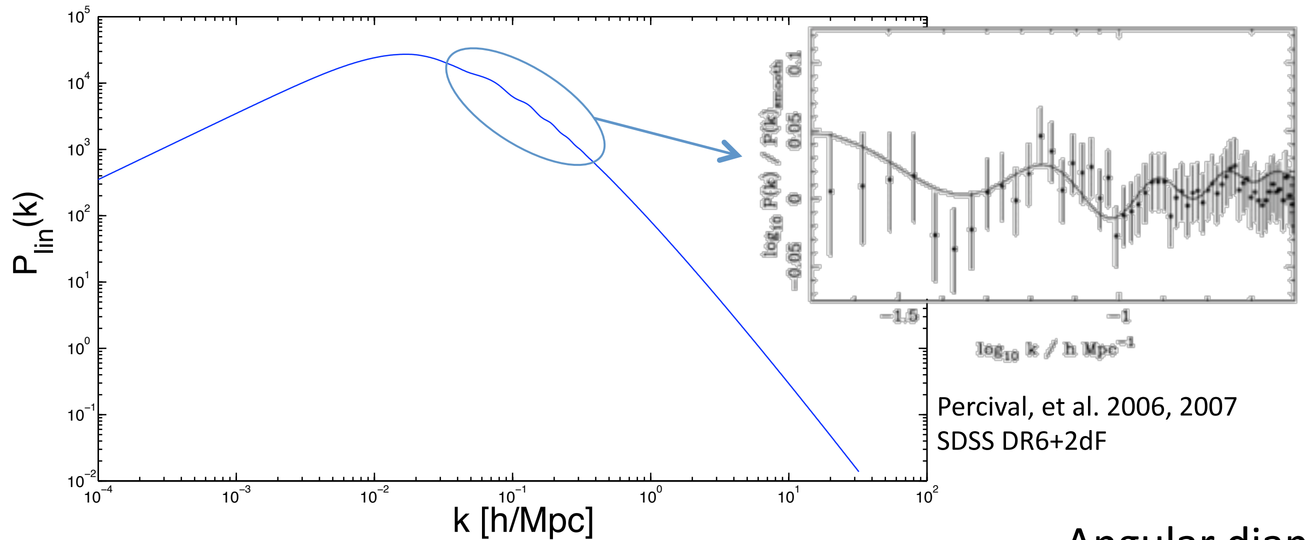
BAO Signal is a “Standard Ruler”



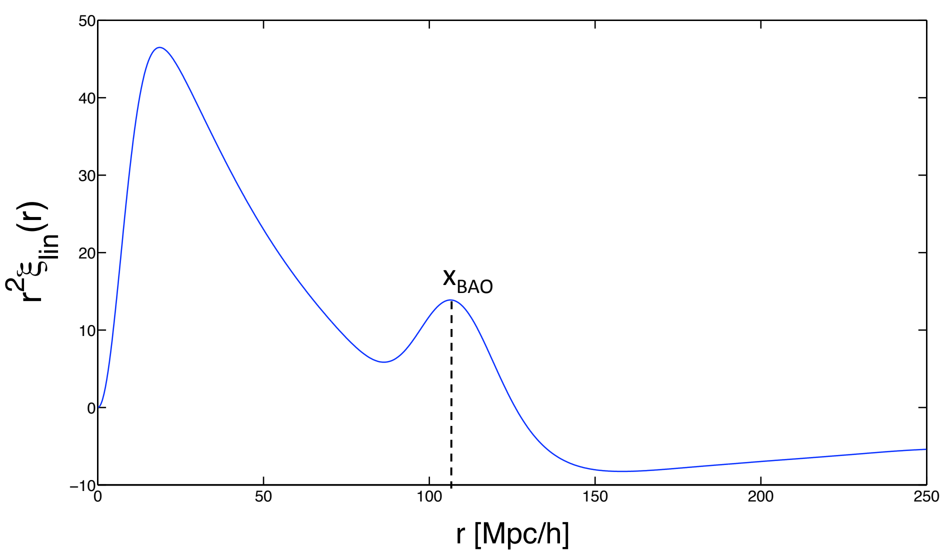
Percival, et al. 2006, 2007
SDSS DR6+2dF

Linear Theory:

$$\delta(\mathbf{x},t) = D(t)\delta_L(\mathbf{x})$$

$$P(k,t) = D(t)^2 P_L(k)$$

$$\xi(r,t) = D(t)^2 \xi_L(r)$$



Angular diameter distance as a function of redshift places constraints on Dark Energy

$$d_A = \frac{x_{BAO}}{\Delta\theta} = \frac{r}{1+z}$$

$$= \frac{1}{1+z}\int_0^z \frac{c\,dz}{H(z)}$$

Nonlinearity and Redshift-space Distortions

•Perturbation theory is used to understand the effects of nonlinearity on quasi-linear scales (Vishniac, 1983)

$$\delta(\mathbf{x},t) = \sum_{n=1}^{\infty} D(t)^n \delta^{(n)}(\mathbf{x})$$

•Redshift measured from Doppler shift, used to calculate distance

- •Galaxies are not at rest in comoving frame

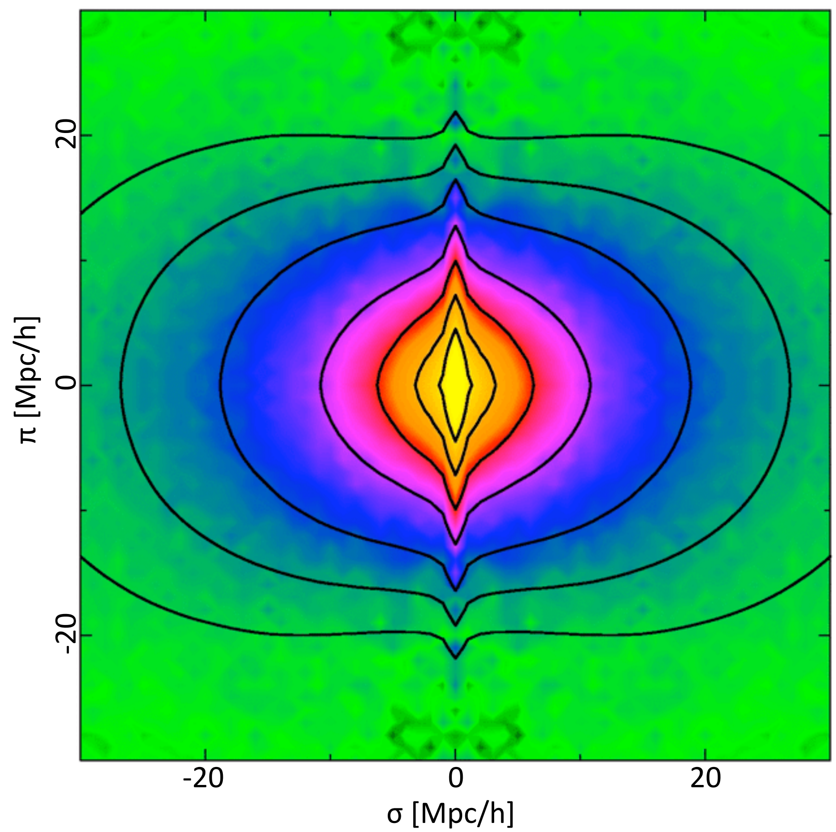
•Linear in-fall (large scales)

- •Flattening of redshift-space correlations

•Thermal motion (small scales)

- •'Fingers of God'

Hawkins et al. (2002), astro-ph/0212375
2dFGRS: β=0.49±0.09



Standard Perturbation Theory

The effect of nonlinearity on the BAO peak is usually studied using perturbation theory in Fourier space (e.g. Jain & Bertschinger, 1994)
Goal is to write the nonlinear power spectrum in terms of the linear/initial quantities.

$$P_{22}(k) = 2\int d^3q\, P_{11}(q)\, P_{11}(|\vec{k}-\vec{q}|)\left[F_2^{(s)}(\vec{q},\vec{k}-\vec{q})\right]^2$$

$$F_2^{(s)}(\vec{k}_1,\vec{k}_2) = \frac{5}{7} + \frac{2}{7}\frac{(\vec{k}_1\cdot\vec{k}_2)^2}{k_1^2k_2^2} + \frac{(\vec{k}_1\cdot\vec{k}_2)}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$$

Motivation for developing perturbation theory in configuration space:

•Structure of the Fourier space kernels suggest that in real space, the result may be simpler
•Real space can be easily extended to redshift space
•It may be simpler to calculate terms beyond 2nd order in configuration space than in Fourier space

New Approach: Perturbation Theory in Configuration Space

We begin with 1st order Lagrangian perturbation theory (Zel'dovich approximation) to verify our approach:

1LPT: $\mathbf{x}(\mathbf{q},t) = \mathbf{q} - D(t)\vec{\nabla}\phi(\mathbf{q})$

$$\nabla^2\Phi(\mathbf{q}) = 4\pi G\bar{\rho}\delta_L(\mathbf{q})$$

$$4\pi G\bar{\rho}\phi(\mathbf{q}) \equiv \Phi(\mathbf{q})$$

$$\frac{\rho(\mathbf{x},t)}{\bar{\rho}} = \left|\frac{\partial x_i}{\partial q_j}\right|^{-1} = \frac{1}{J(\mathbf{q},t)} = 1 + \delta(\mathbf{q}(\mathbf{x}))$$

Expansion of the density in terms of the growth function:

$$\delta(\mathbf{x},t) = \left(\delta(\mathbf{q},t) + D\sum_i \frac{\partial\phi(\mathbf{q})}{\partial q_i}\frac{\partial\delta(\mathbf{q},t)}{\partial q_i} + D^2\sum_{i,j}\frac{\partial^2\phi(\mathbf{q})}{\partial q_i\partial q_j}\frac{\partial\phi(\mathbf{q})}{\partial q_j}\frac{\partial\delta(\mathbf{q},t)}{\partial q_i} + \frac{1}{2}D^2\sum_{i,j}\frac{\partial^2\delta(\mathbf{q},t)}{\partial q_i\partial q_j}\frac{\partial\phi(\mathbf{q})}{\partial q_i}\frac{\partial\phi(\mathbf{q})}{\partial q_j}\right)\Bigg|_{\mathbf{q}=\mathbf{x}}.$$

New Approach: Perturbation Theory in Configuration Space

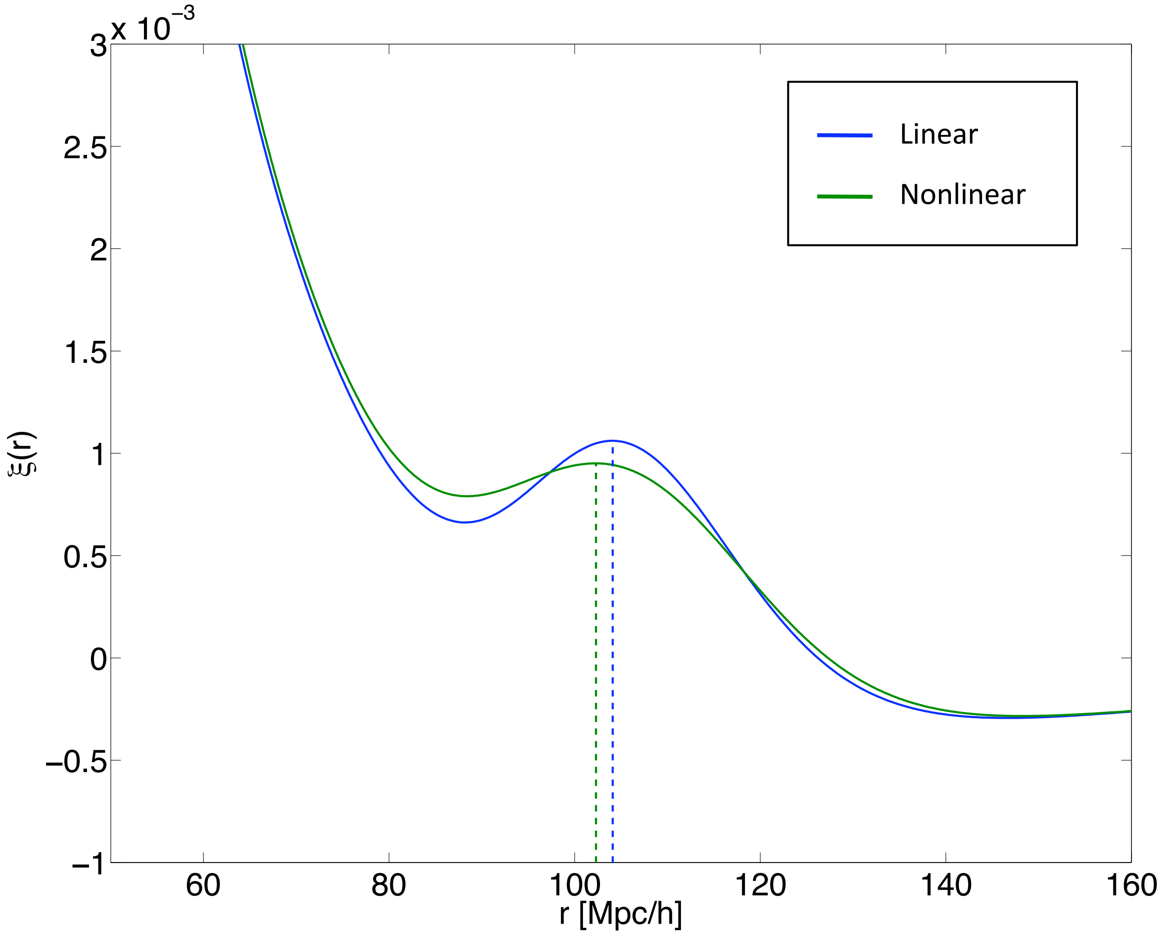
$$\xi(\mathbf{x}_1 - \mathbf{x}_2, t) = \langle \delta(\mathbf{x}_1, t)\delta(\mathbf{x}_2, t)\rangle$$

$$\xi(\mathbf{r}, t) = \xi^{(1)}(\mathbf{r})D^2 + \xi^{(2)}(\mathbf{r})D^4 + \ldots$$

$$\boxed{\xi_n^m(r) = \frac{1}{2\pi^2}\int P_L(k) j_n(kr) k^{m+2} dk}$$

$$\boxed{\begin{aligned}\xi^{(2)}(\mathbf{r}) = &-\frac{1}{3}\xi_0^{-2}(0)\xi_0^2(r) + \frac{19}{15}\xi_0^0(r)^2 + \frac{34}{21}\xi_2^0(r)^2 + \frac{4}{35}\xi_4^0(r)^2 - \frac{16}{5}\xi_1^{-1}(r)\xi_1^1(r)\\ &-\frac{4}{5}\xi_3^{-1}(r)\xi_3^1(r) + \frac{1}{3}\xi_0^{-2}(r)\xi_0^2(r) + \frac{2}{3}\xi_2^{-2}(r)\xi_2^2(r)\end{aligned}}$$

The Effect of the Nonlinear Term on the Acoustic Peak



At z=0, the peak is damped by about 10% and shifted to lower r by about 1%

Comparison to Fourier Space

$$\xi^{(2)}(\mathbf{r}) = -\frac{1}{3}\xi_0^{-2}(0)\xi_0^2(r) + \frac{19}{15}\xi_0^0(r)^2 + \frac{34}{21}\xi_2^0(r)^2 + \frac{4}{35}\xi_4^0(r)^2 - \frac{16}{5}\xi_1^{-1}(r)\xi_1^1(r) - \frac{4}{5}\xi_3^{-1}(r)\xi_3^1(r) + \frac{1}{3}\xi_0^{-2}(r)\xi_0^2(r) + \frac{2}{3}\xi_2^{-2}(r)\xi_2^2(r)$$

vs

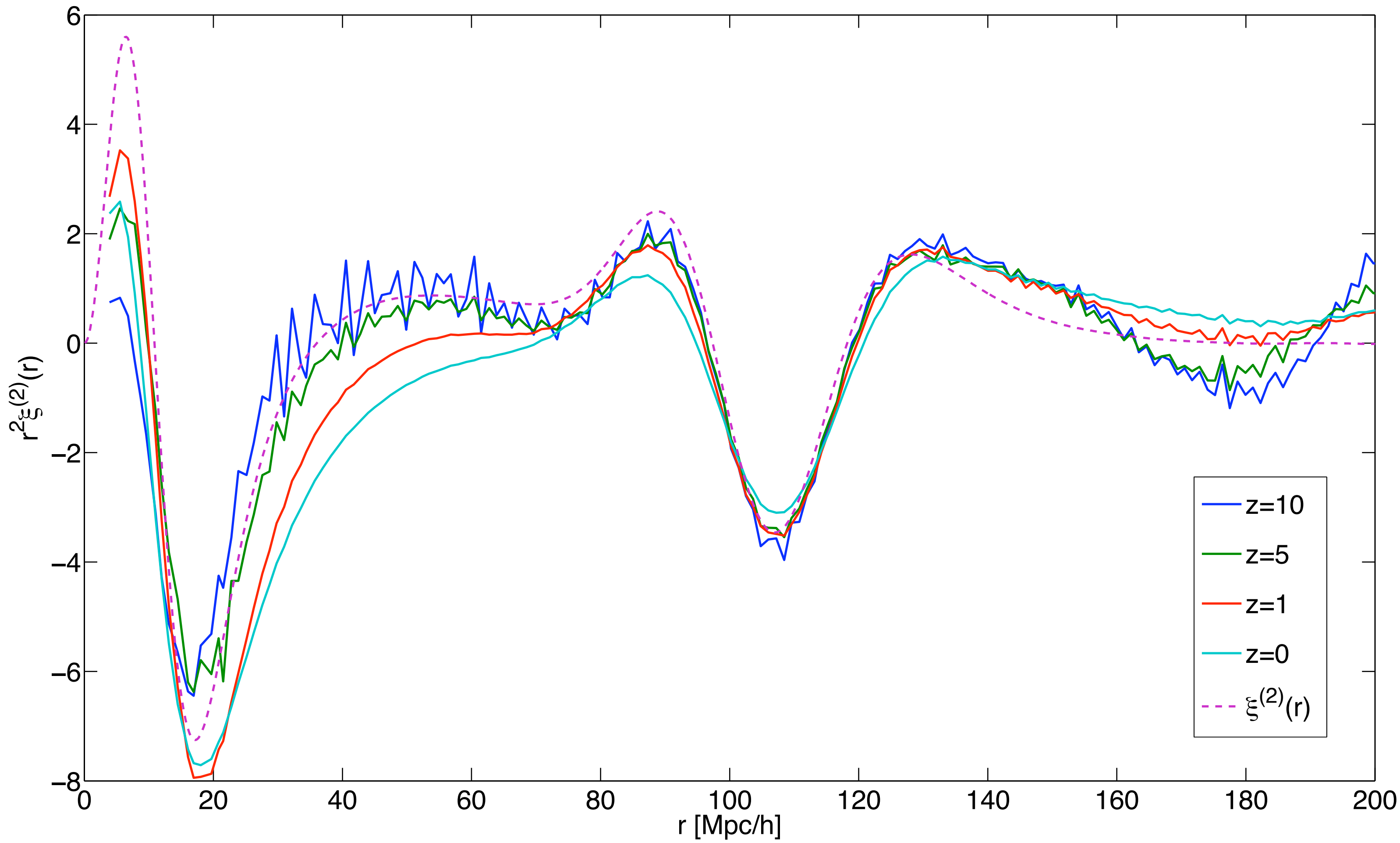
$$P^{(2)}(k) = -k^2\sigma_v^2 P_L(k) + \int\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}\cdot\mathbf{k}_1)^2(\mathbf{k}\cdot\mathbf{k}_2)^2}{2k_1^4k_2^4}P_L(k_1)P_L(k_2)$$

$$\sigma_v^2 = \frac{1}{6\pi^2}\int P_L(w)dw$$

From Valageas, 2011

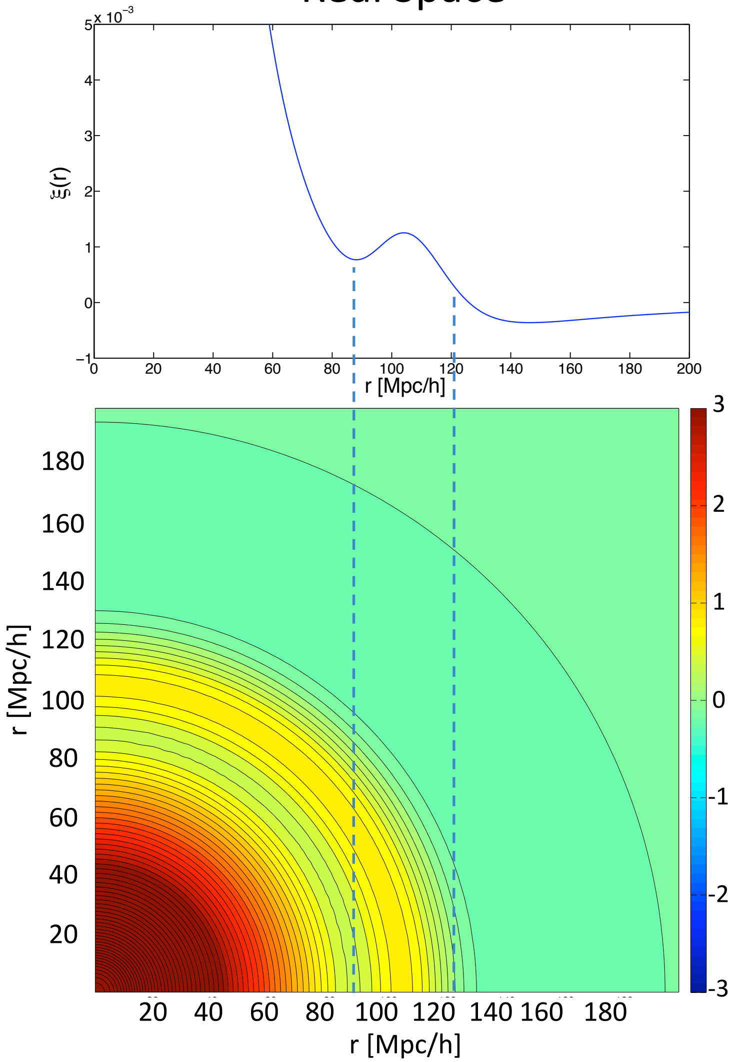
Full calculation in arXiv:1202.1306v2

Comparison to Numerical Simulation



Correlation Function in Real and Redshift Space

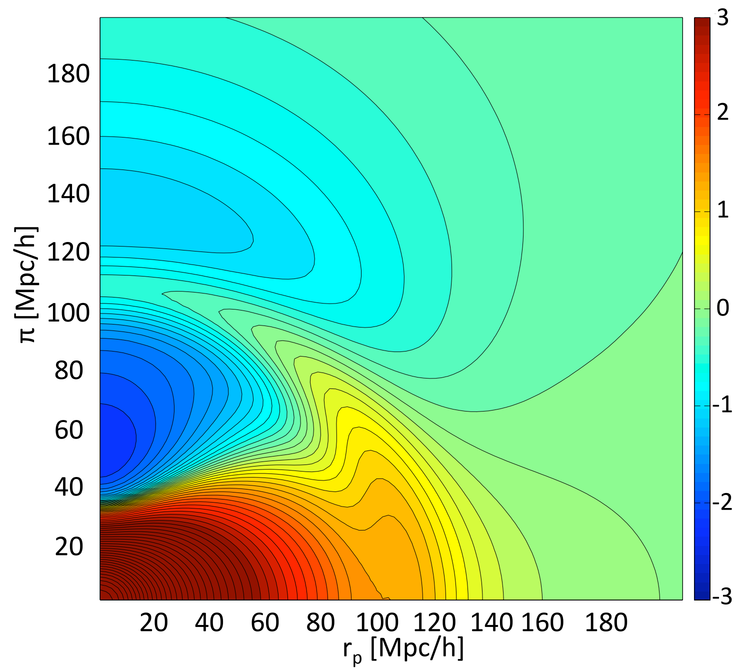
Real Space



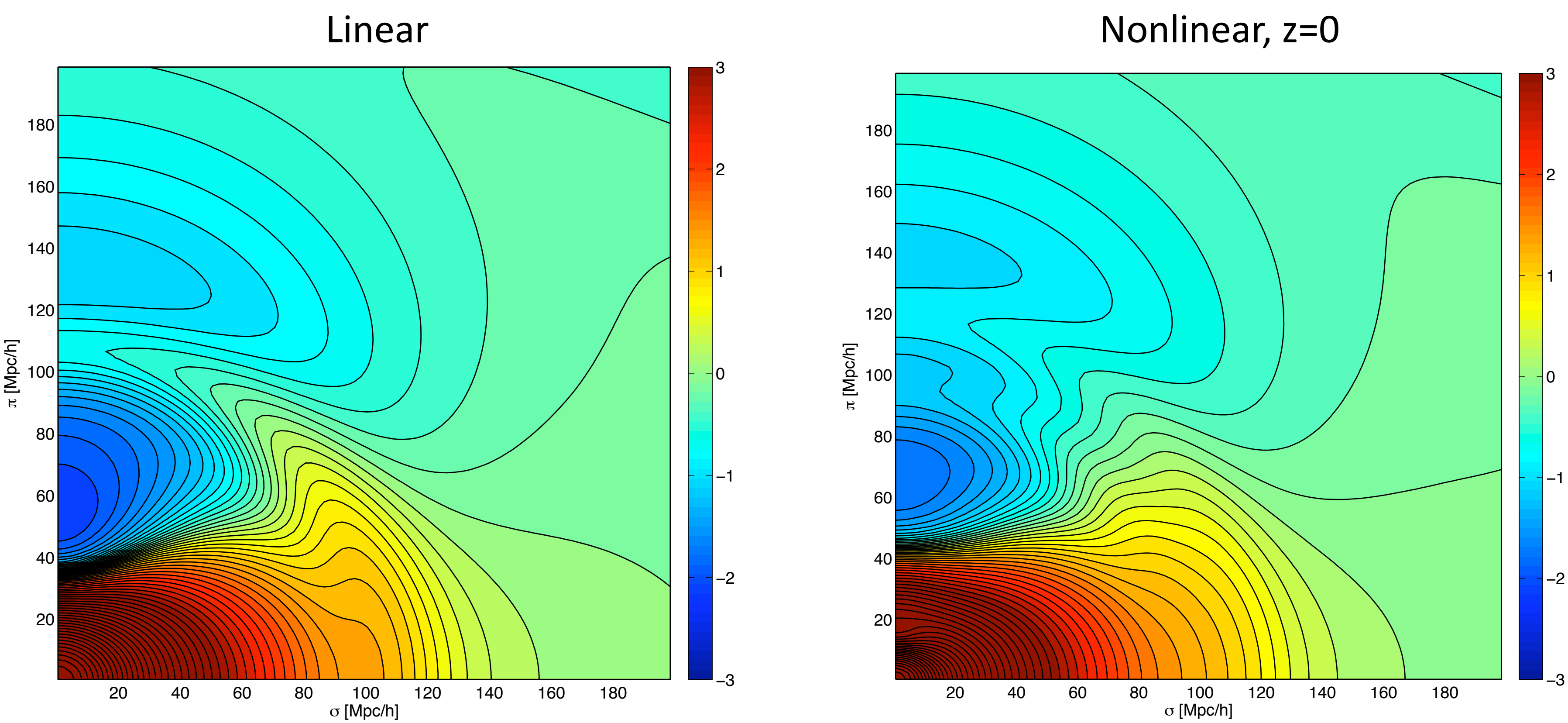
$$\mathbf{s} = \mathbf{x} - u_z(\mathbf{x})\hat{\mathbf{z}}$$

$$\xi^{(1)}(\mathbf{s}) = \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\xi_0^0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mathcal{P}_2(\mu)\xi_2^0(s) + \frac{8f^2}{35}\mathcal{P}_4(\mu)\xi_4^0(s)$$

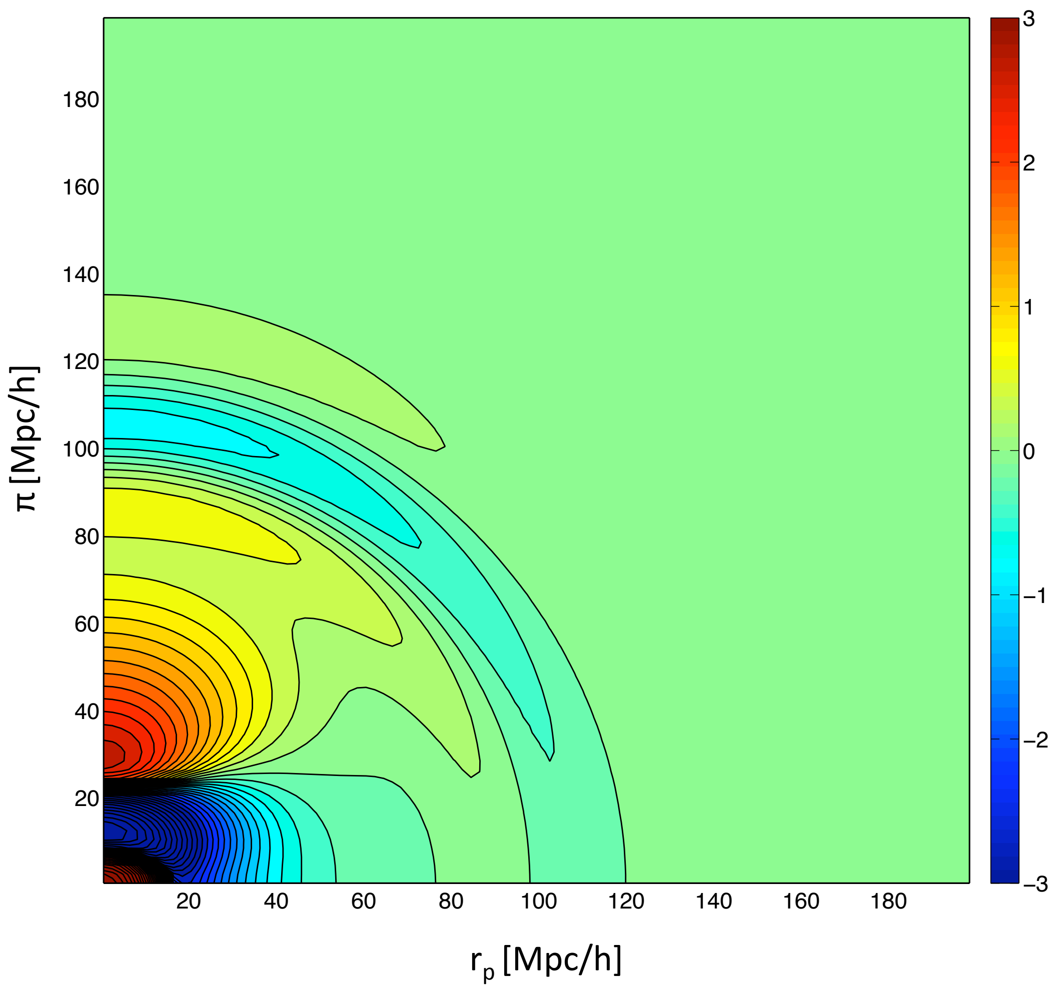
Redshift Space

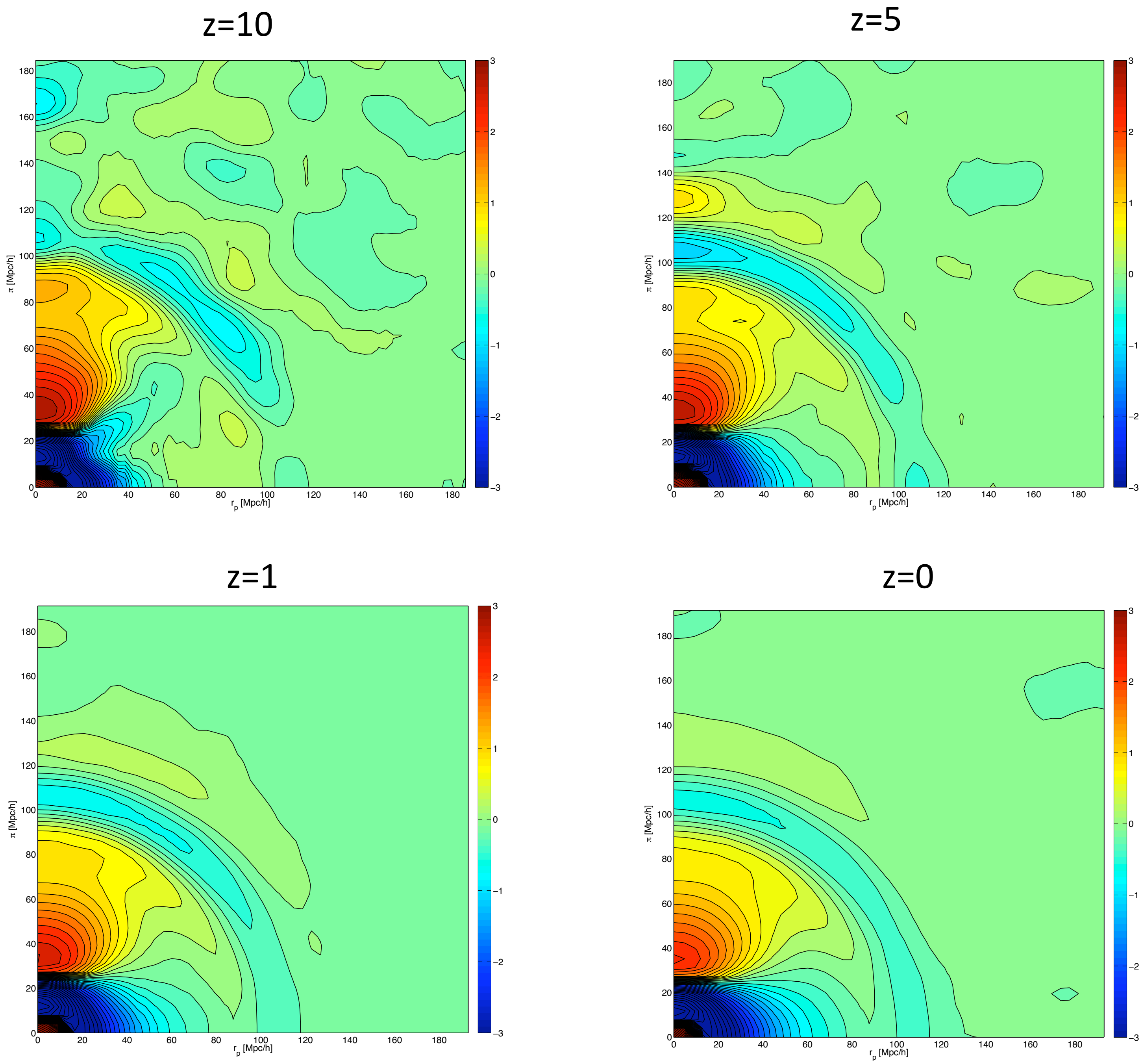


Nonlinear Correlation Function in Redshift Space



Nonlinear Correlation Function in Redshift Space




z=10
z=5
z=1
z=0
π [Mpc/h]
r_p [Mpc/h]
0
20
40
60
80
100
120
140
160
180
3
2
1
0
−1
−2
−3

Future Work

Higher orders in Lagrangian Perturbation Theory (2LPT, etc) give the correct higher order behavior in the density

- Extension to redshift space
- Comparison to full N-body simulations (Indra)

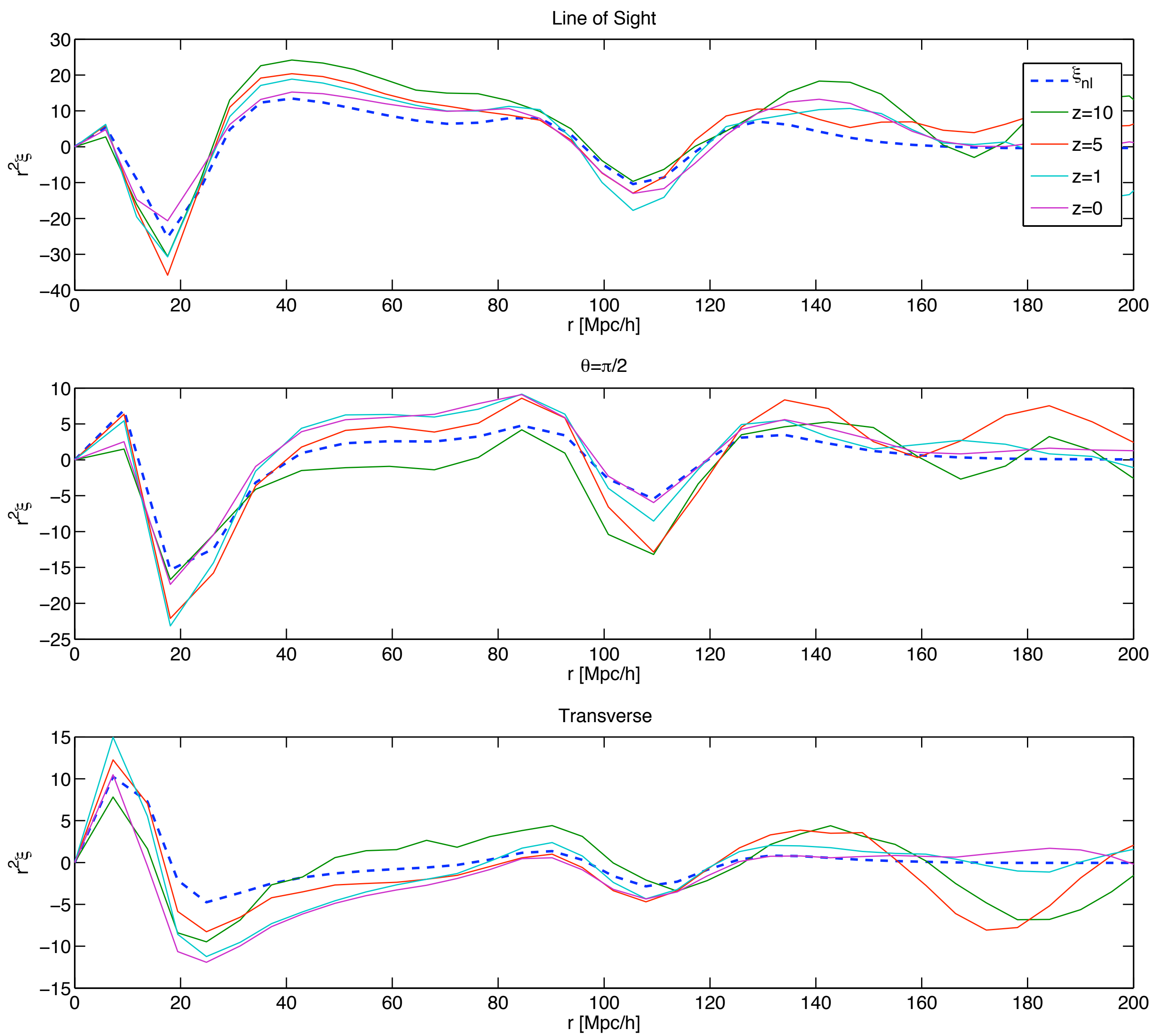
2LPT: $\vec{x} = \vec{q} - D_1 \nabla_q \phi^{(1)} + D_2 \nabla_q \phi^{(2)}$

$$\boxed{\begin{aligned}\xi_{(22)}(\vec{r}) \;=\; & \frac{1219}{735}\xi_0^0(r)^2 + \frac{1}{3}\xi_0^{-2}(r)\xi_0^2(r) - \frac{124}{35}\xi_1^{-1}(r)\xi_1^1(r) + \frac{1342}{1029}\xi_2^0(r)^2 + \\ & \frac{2}{3}\xi_2^{-2}(r)\xi_2^2(r) - \frac{16}{35}\xi_3^{-1}(r)\xi_3^1(r) + \frac{64}{1715}\xi_4^0(r)^2\end{aligned}}$$

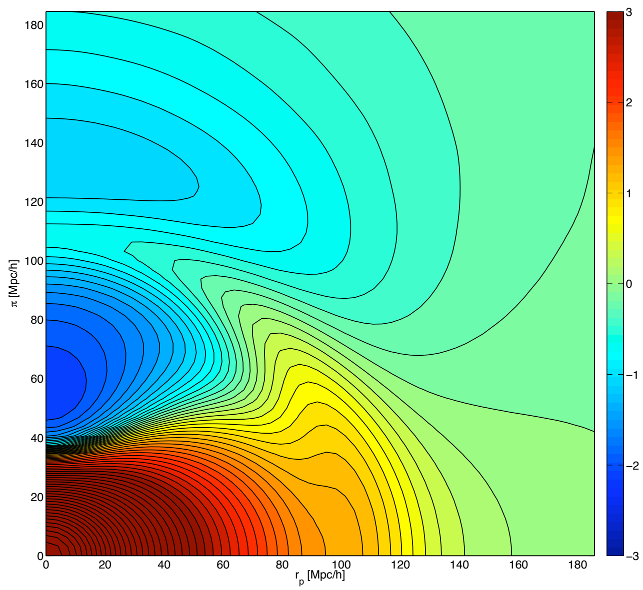
$$P_{22}(k) = 2\int d^3q\, P_{11}(q)\, P_{11}(|\vec{k}-\vec{q}|)\left[F_2^{(s)}(\vec{q},\vec{k}-\vec{q})\right]^2$$

Conclusion

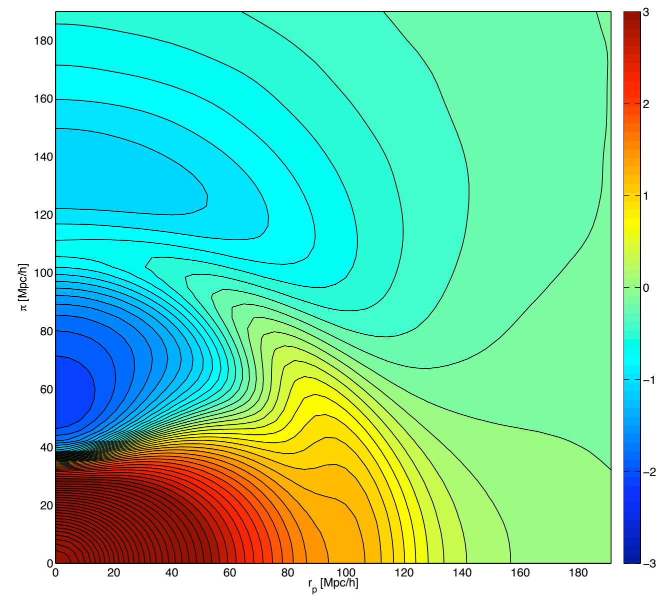
- Our approach to perturbation theory in configuration space works
  - Reproduces known Fourier-space result for Zel'dovich Approximation
  - Agrees with numerical simulations
  - See arXiv:1202.1306v2 for more details
- We can extend this calculation to redshift space
  - Numerical validation of analytical result
- In future, we will extend to higher orders in Lagrangian Perturbation Theory to reproduce correct higher-order behavior
  - Compare to full N-body simulations


Line of Sight
$r^2\xi$
r [Mpc/h]
ξ_{nl}
z=10
z=5
z=1
z=0
θ=π/2
$r^2\xi$
r [Mpc/h]
Transverse
$r^2\xi$
r [Mpc/h]

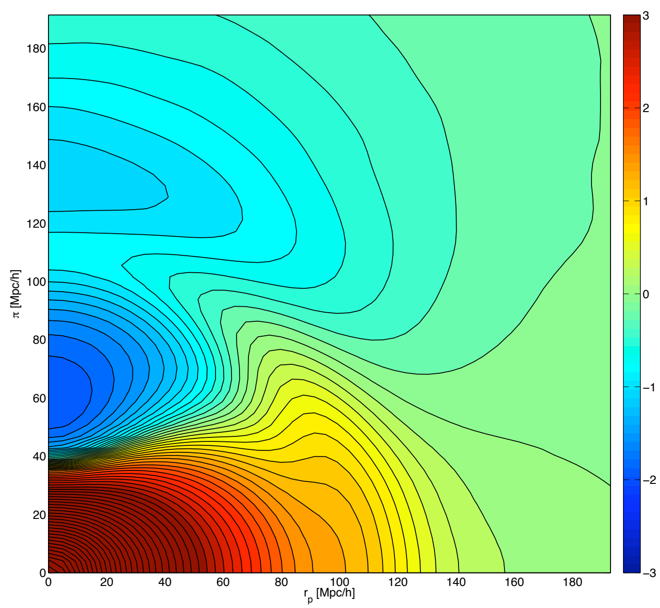
z=10



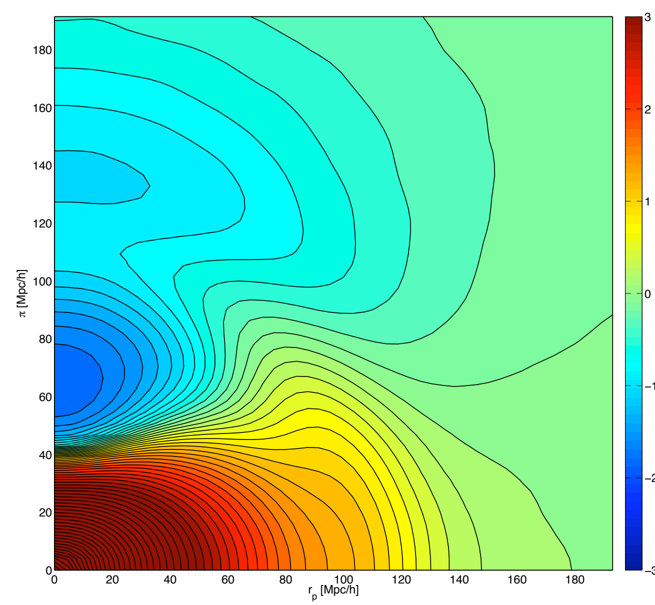
z=5



z=1



z=0



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Redshift-Space Distortions

•Redshift measured from Doppler shift, used to calculate distance
    •Galaxies are not at rest in comoving frame
•Linear in-fall (large scales)
    •Flattening of redshift-space correlations
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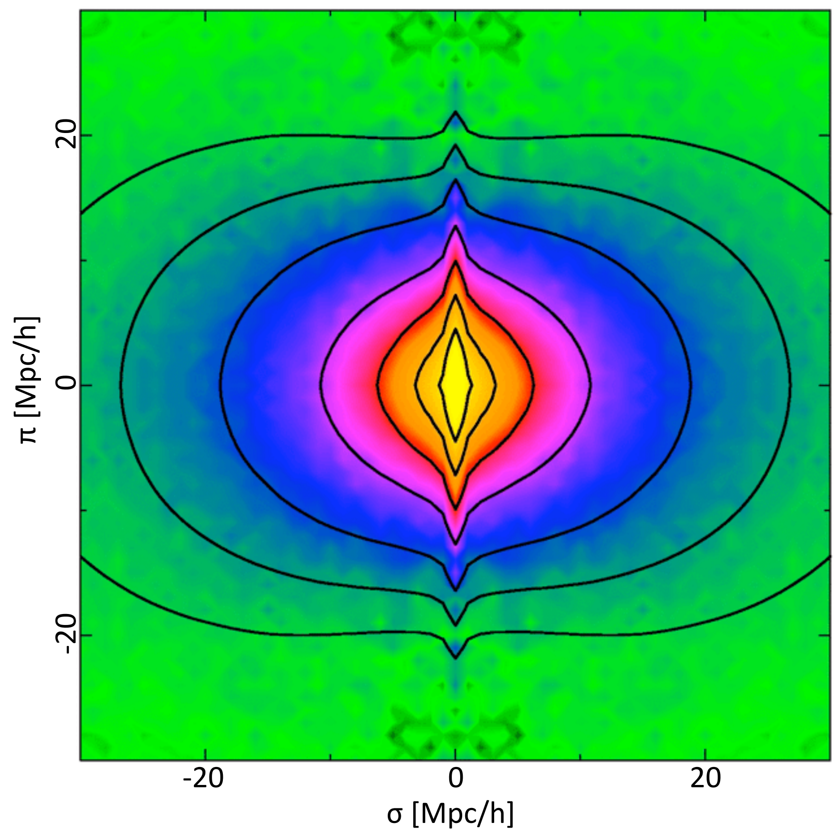

Nonlinear Structure Formation

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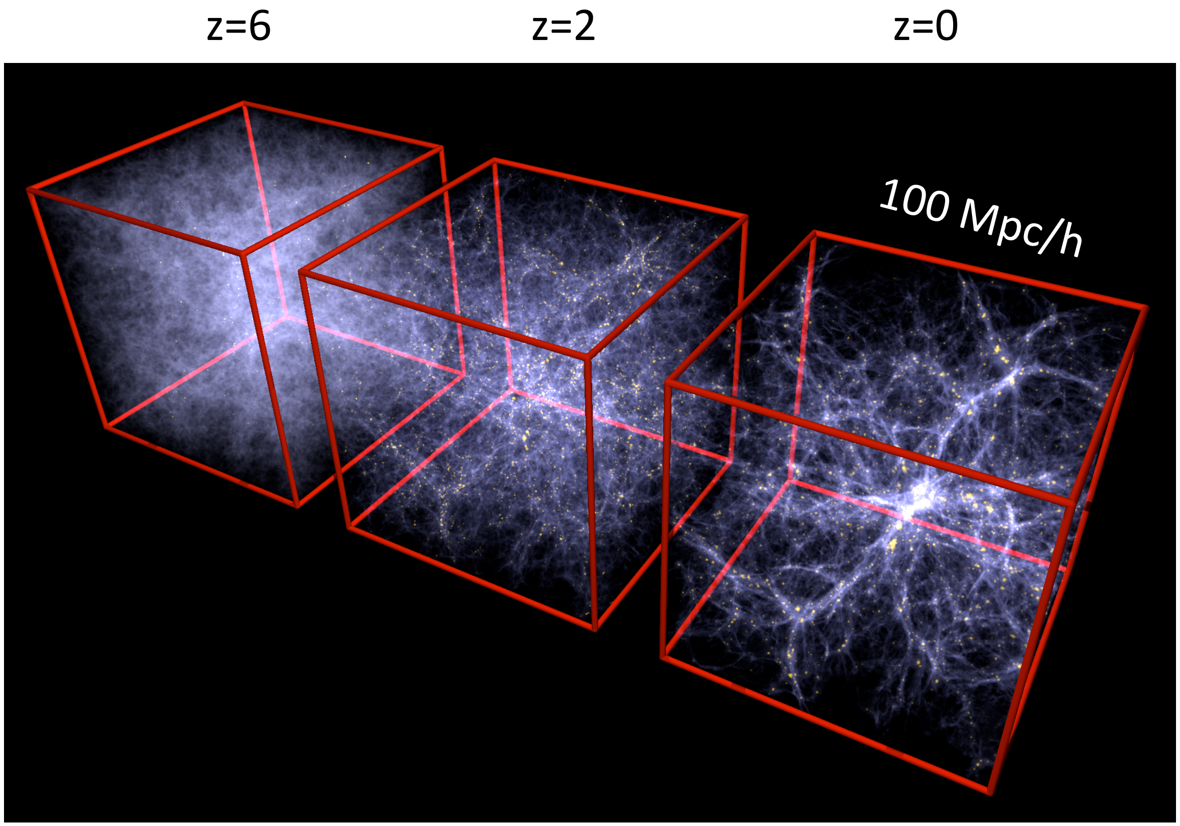
Configuration Space:

$$\delta(\mathbf{x},t) = \sum_{n=1}^{\infty} D(t)^n \delta^{(n)}(\mathbf{x})$$

Fourier Space:

$$\hat{\delta}(\mathbf{k},t) = \sum_{n=1}^{\infty} D(t)^n \hat{\delta}^{(n)}(\mathbf{k})$$



Volker Springel, Max-Planck-Institute for Astrophysics